Verifiable Elections That Scale for Free

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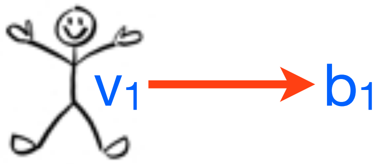

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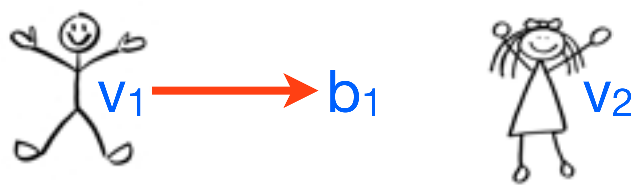

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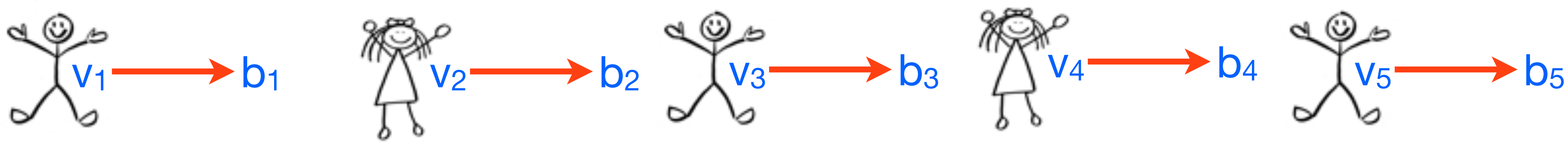

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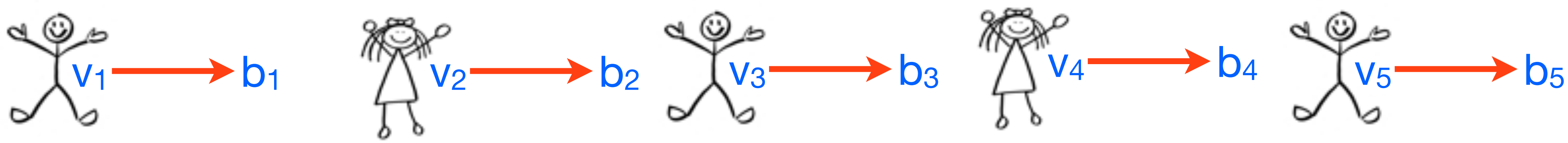

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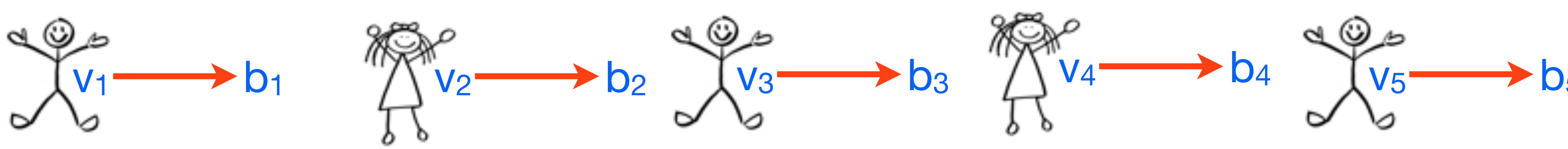

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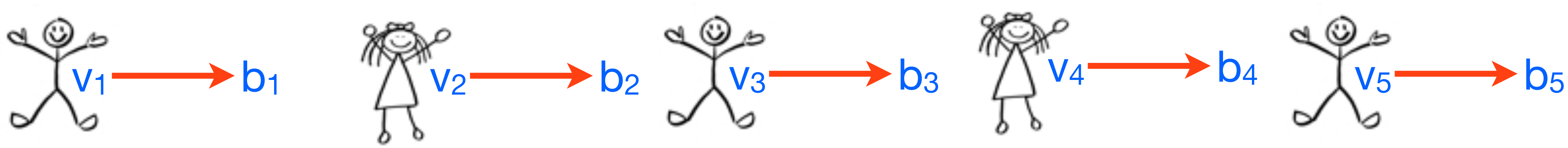

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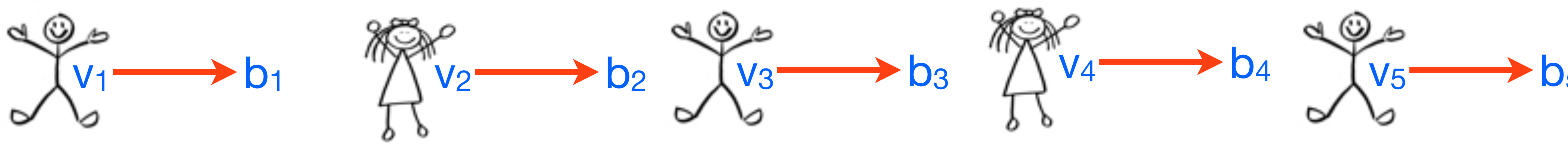

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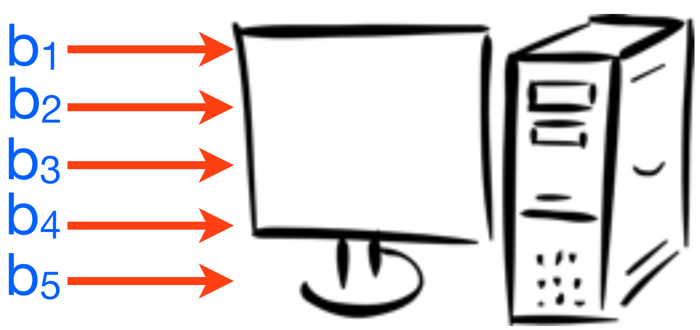

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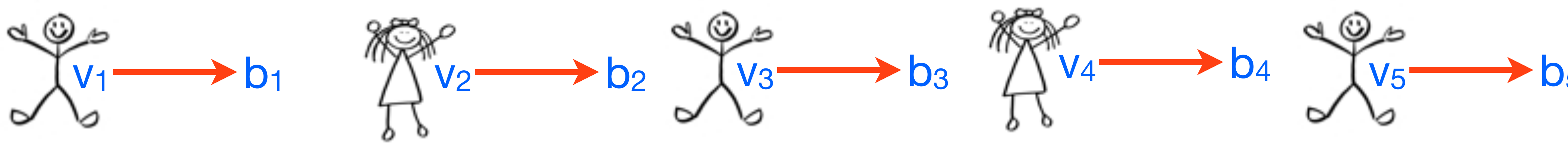

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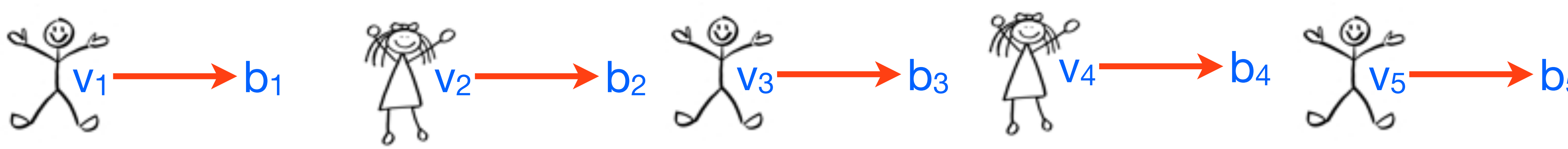

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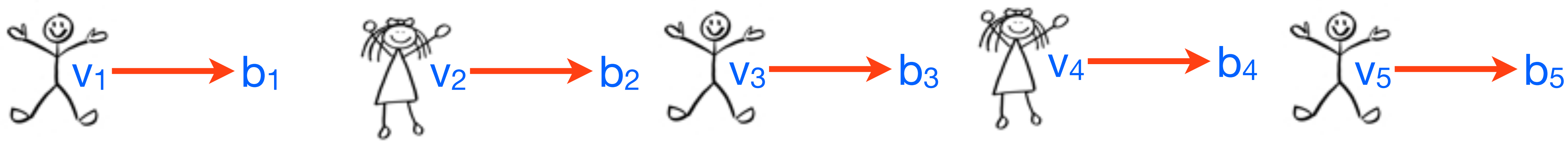

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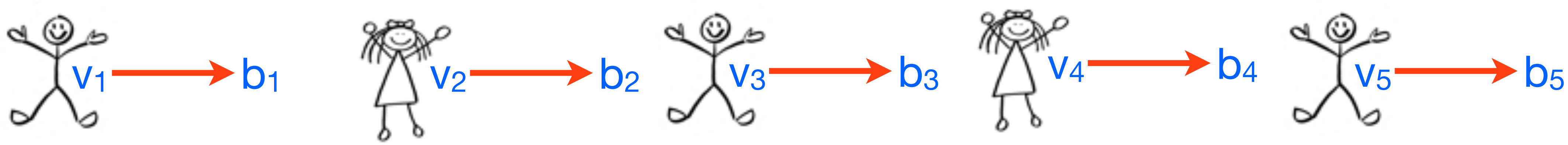

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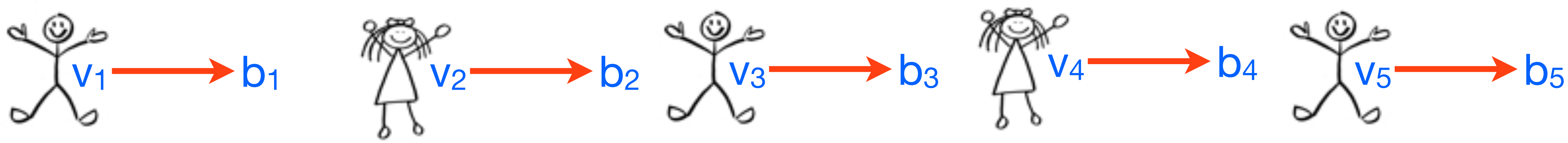

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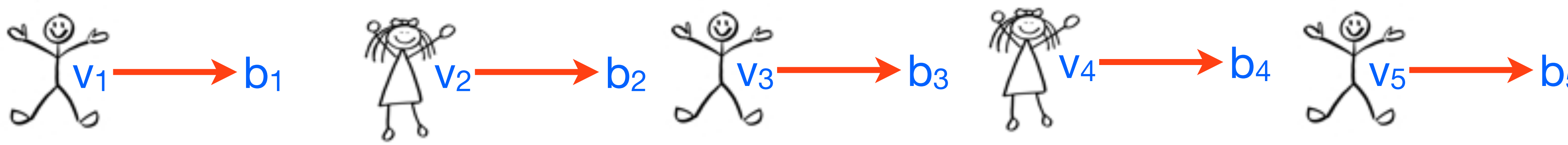

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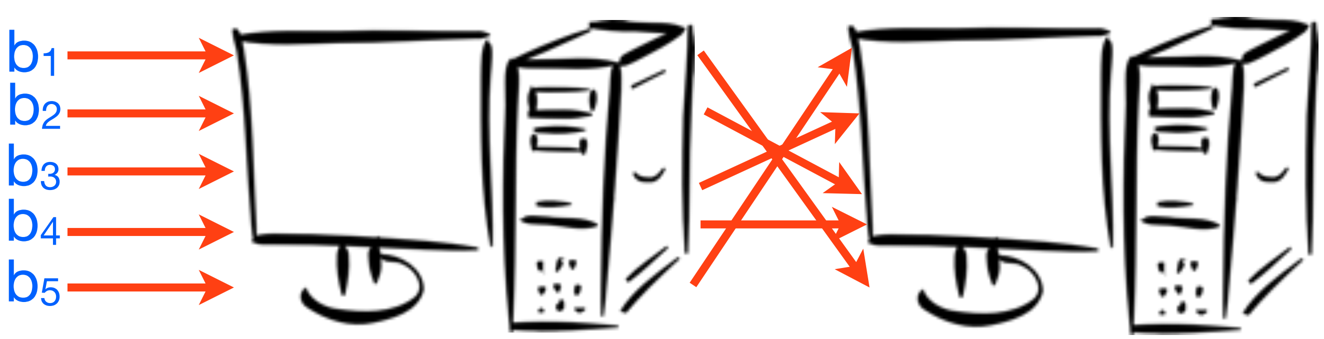

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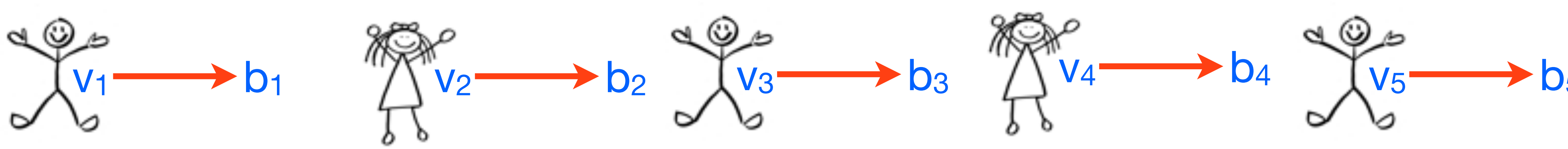

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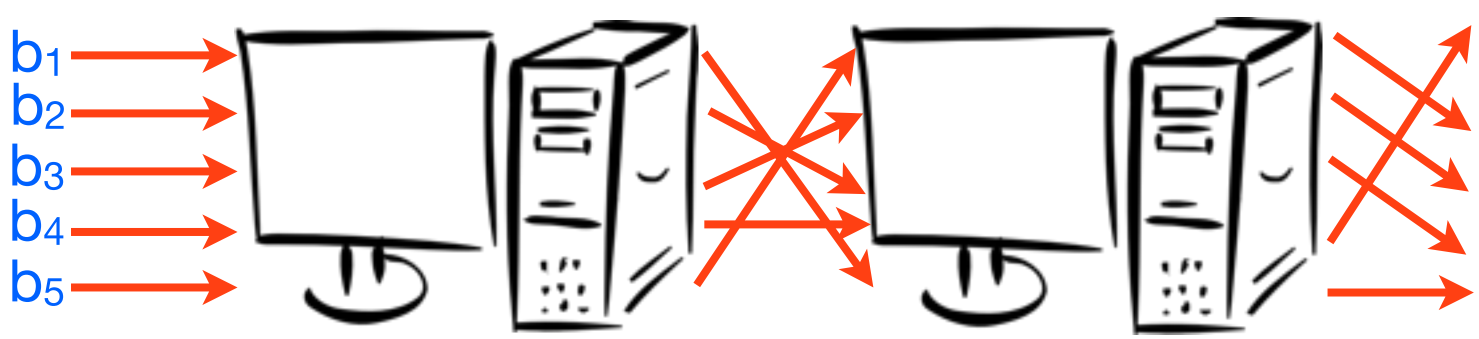

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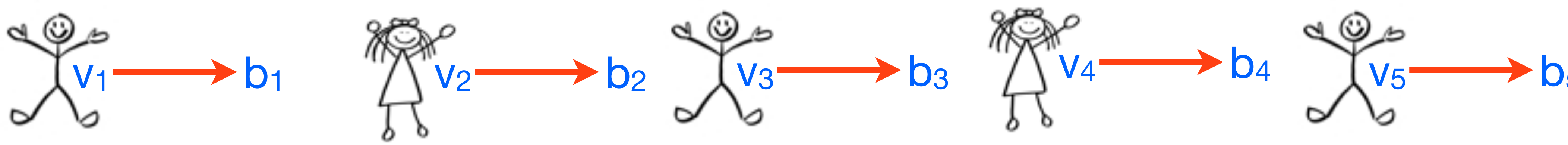

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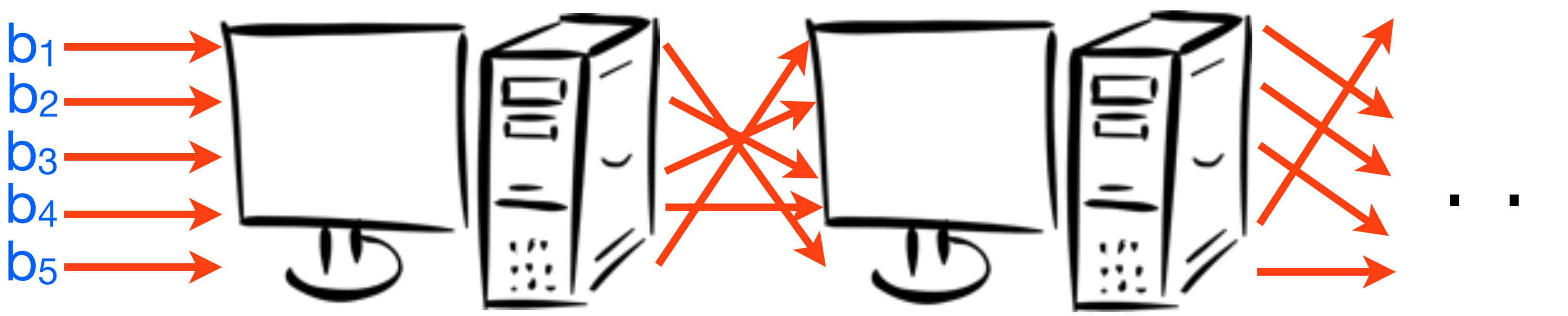

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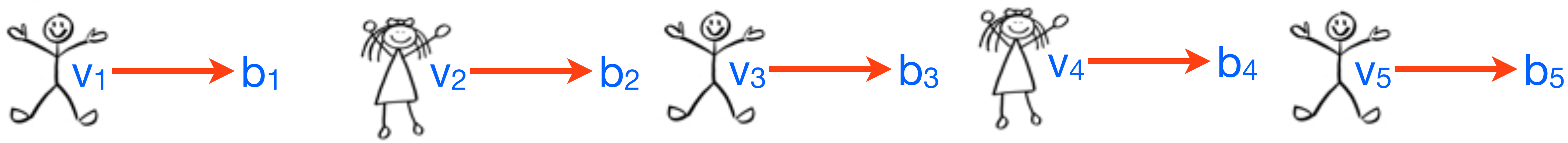

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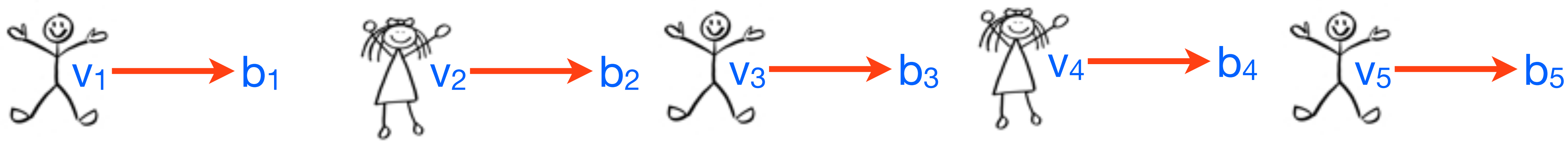

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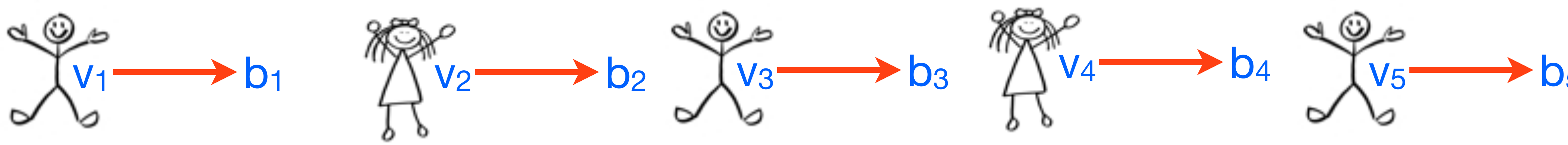

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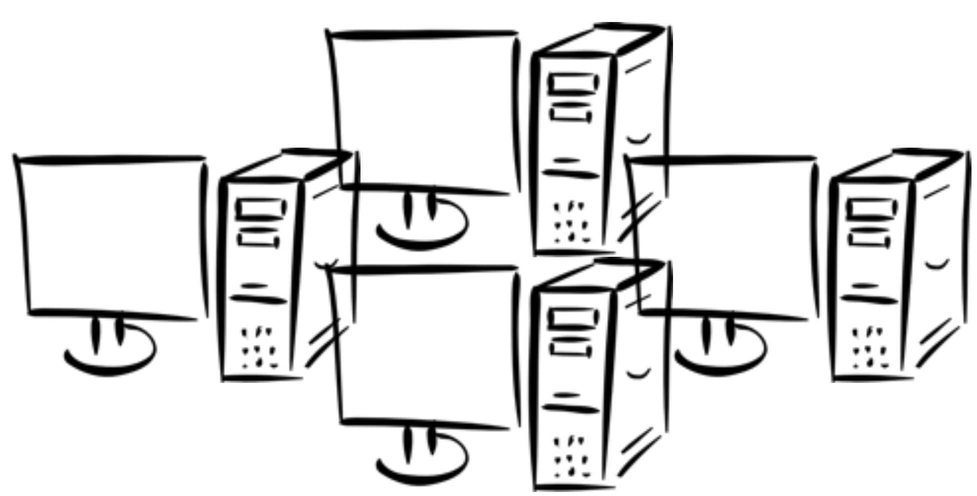

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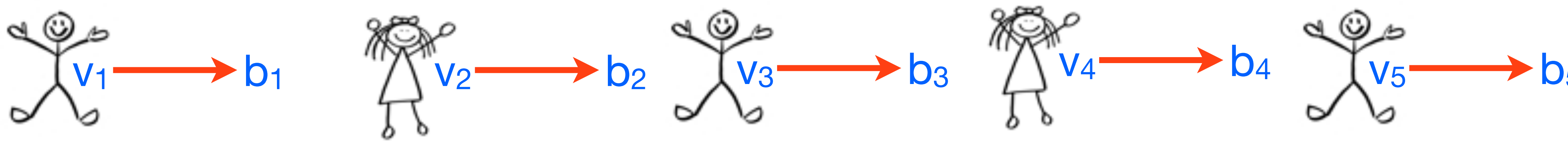

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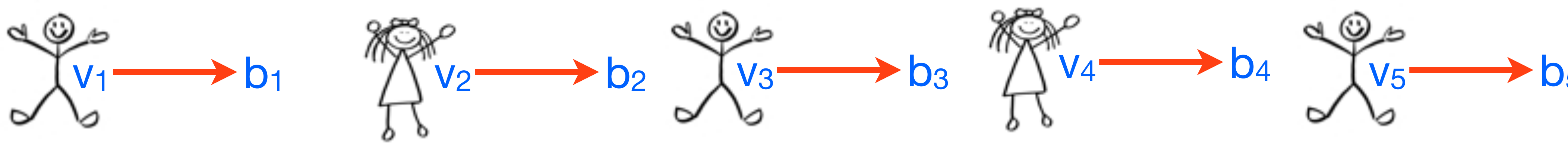

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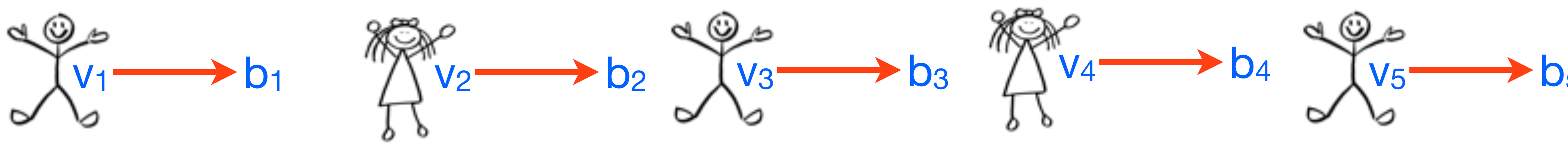

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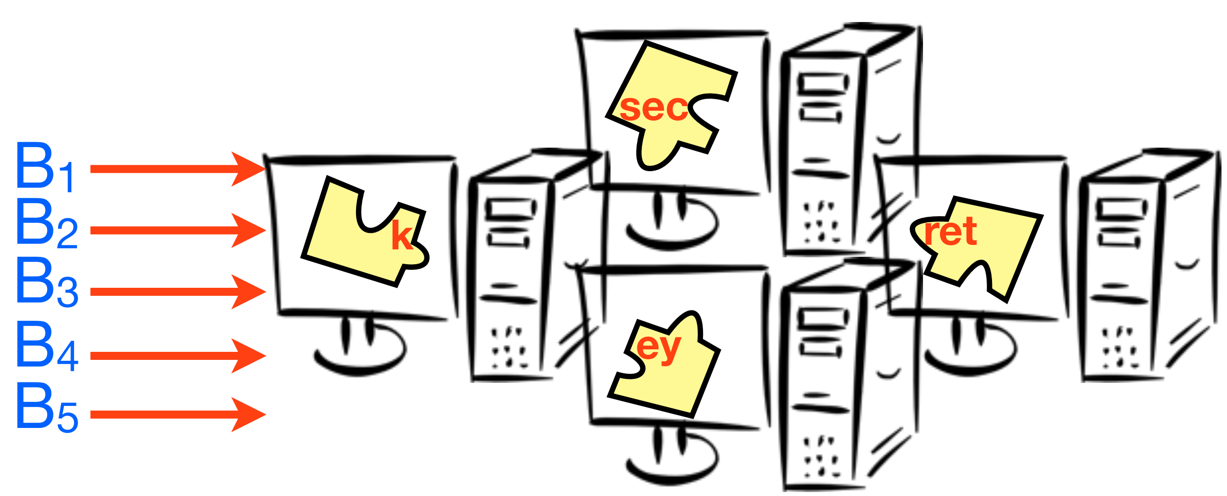

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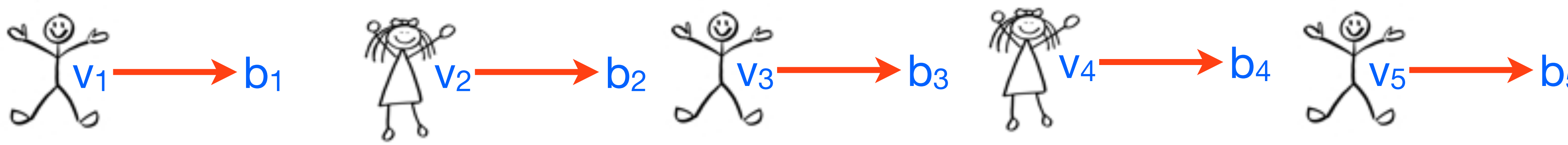

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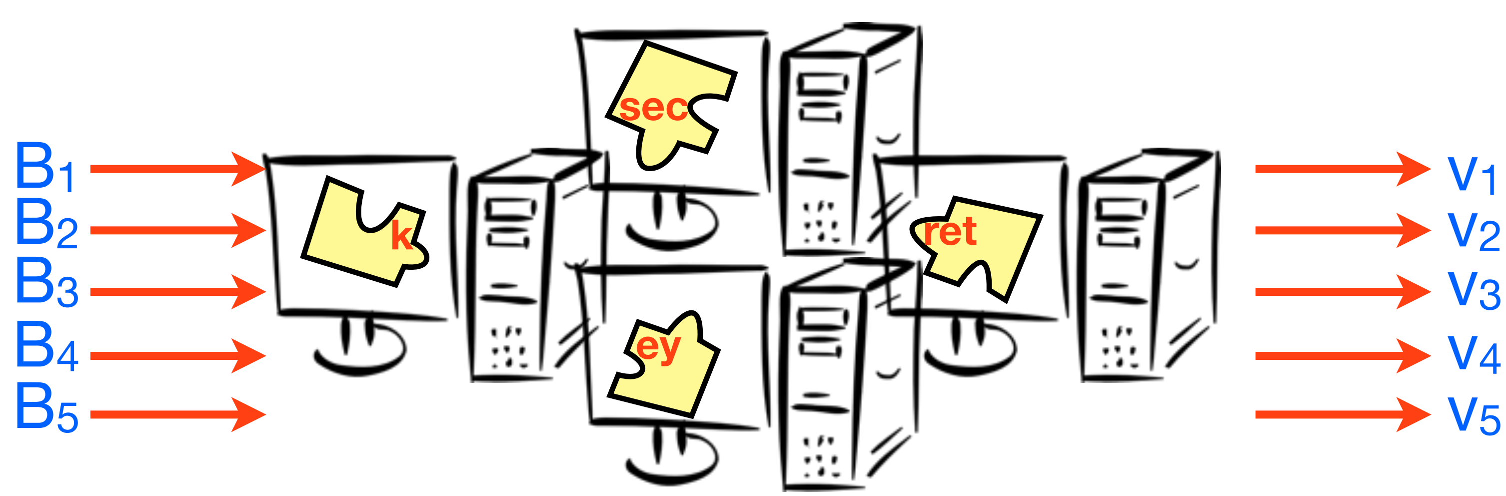

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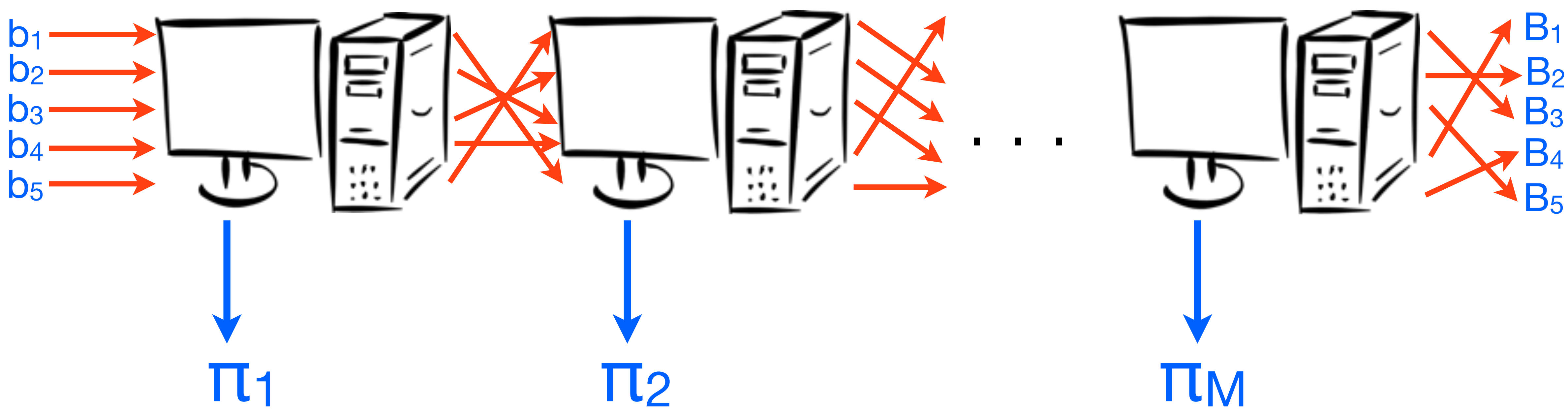

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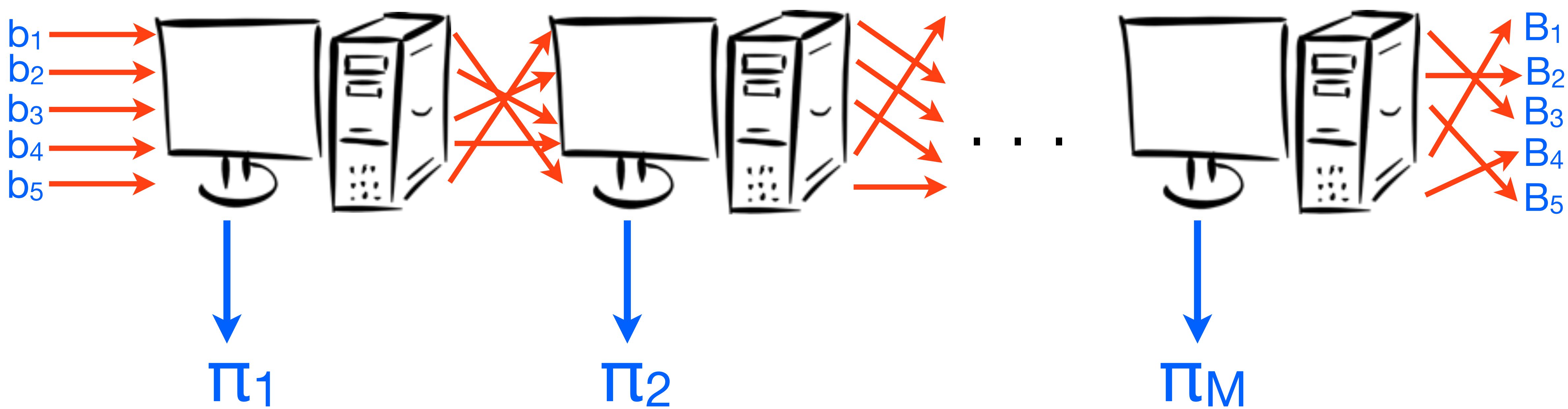

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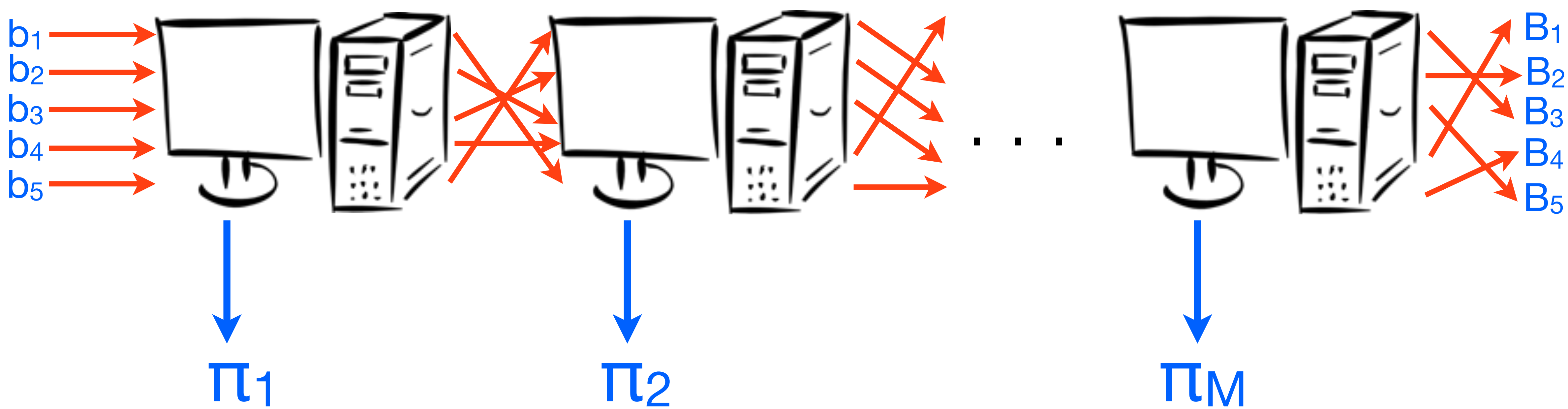

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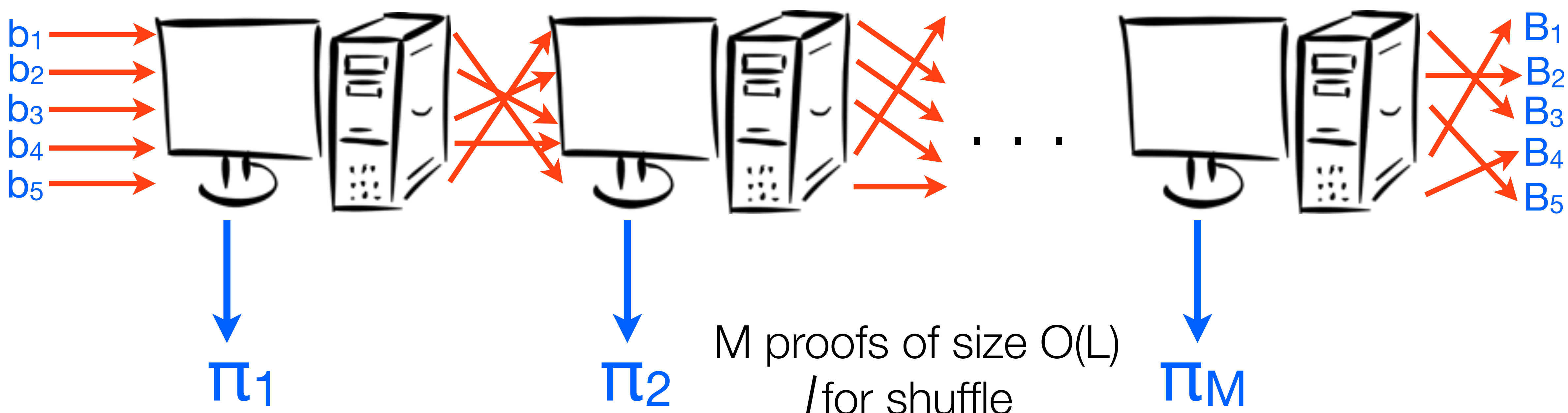

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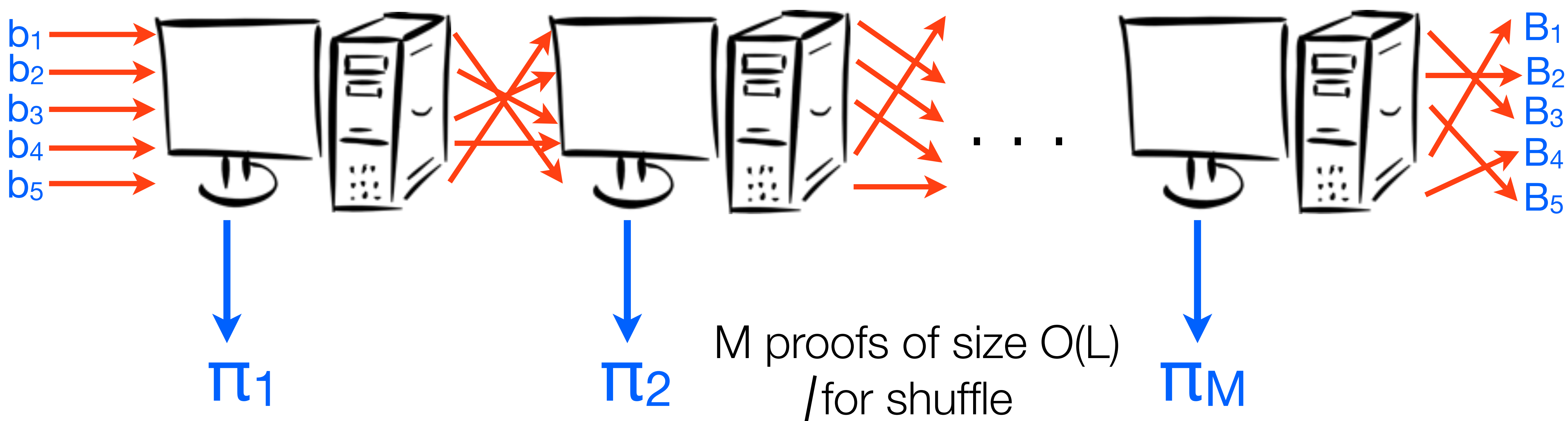

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N proofs of size O(L)
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- Give efficient instantiations of shuffle and threshold decryption schemes based on Decision Linear [BBS04] and two static assumptions [GL07]

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But how to define a strong notion of soundness like extractability?

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If a proof is zero knowledge, CM-SSE, and strongly derivation private, then we call it a cm-NIZK

(like function privacy for homomorphic encryption)

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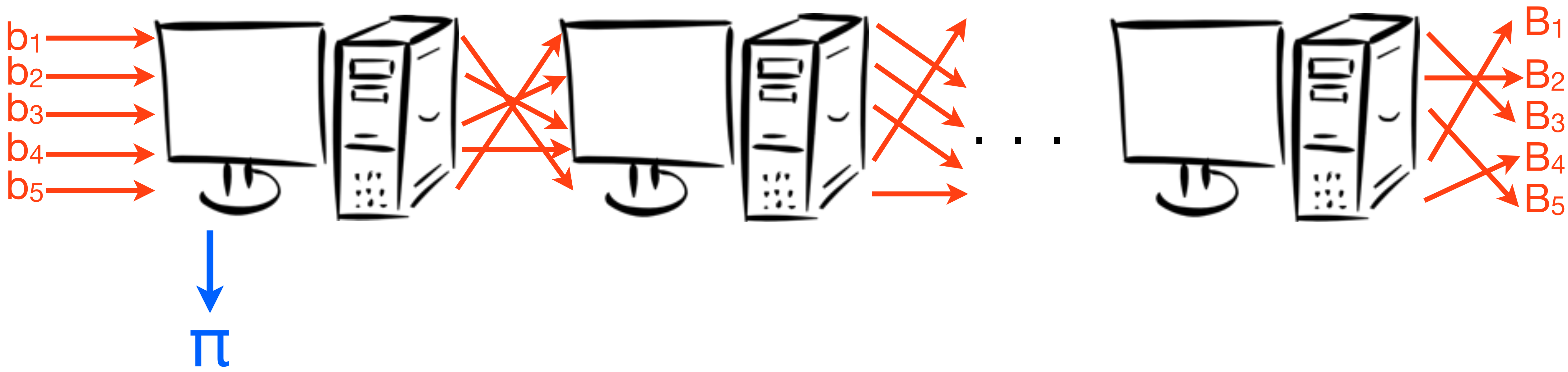

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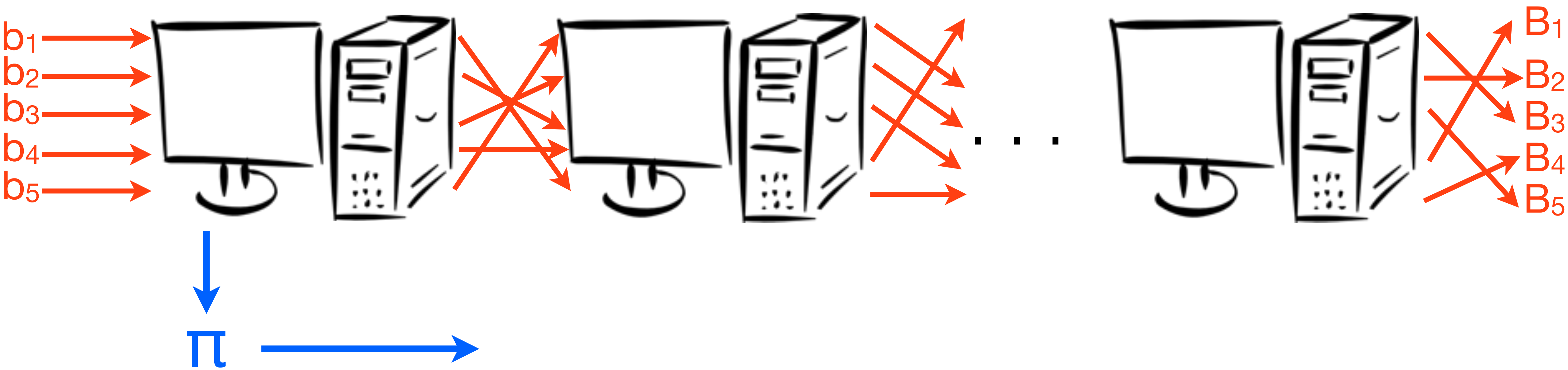

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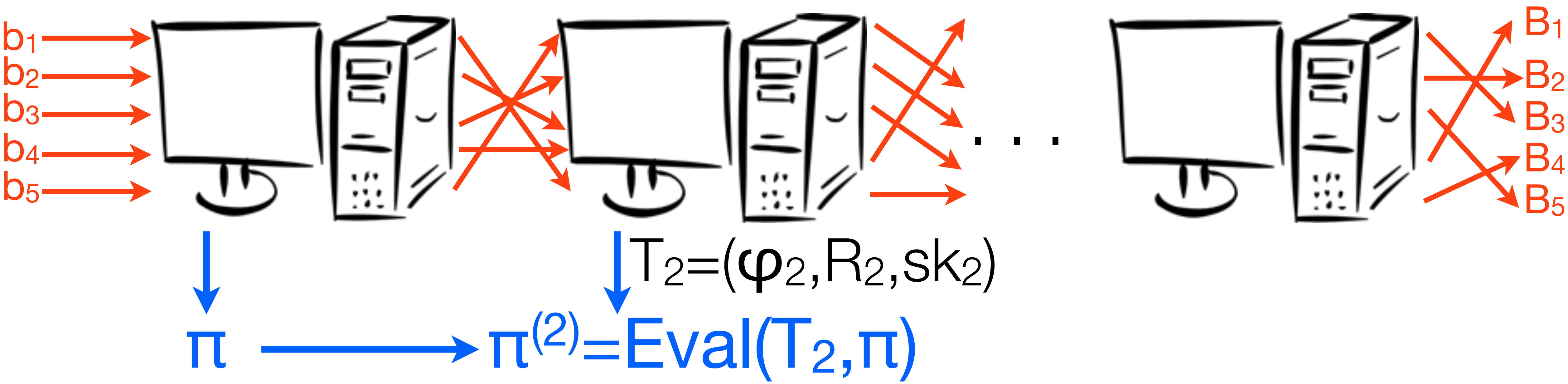

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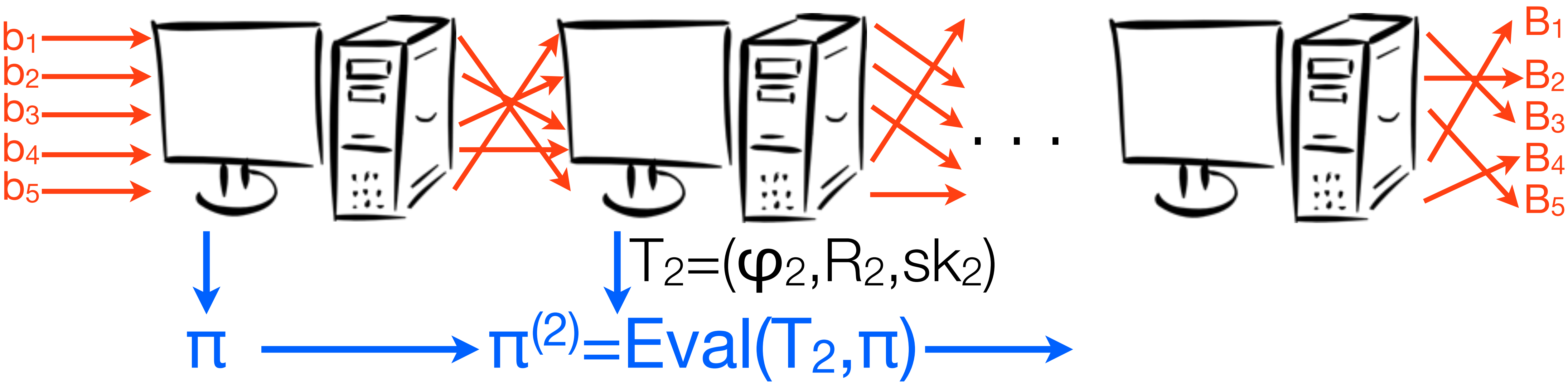

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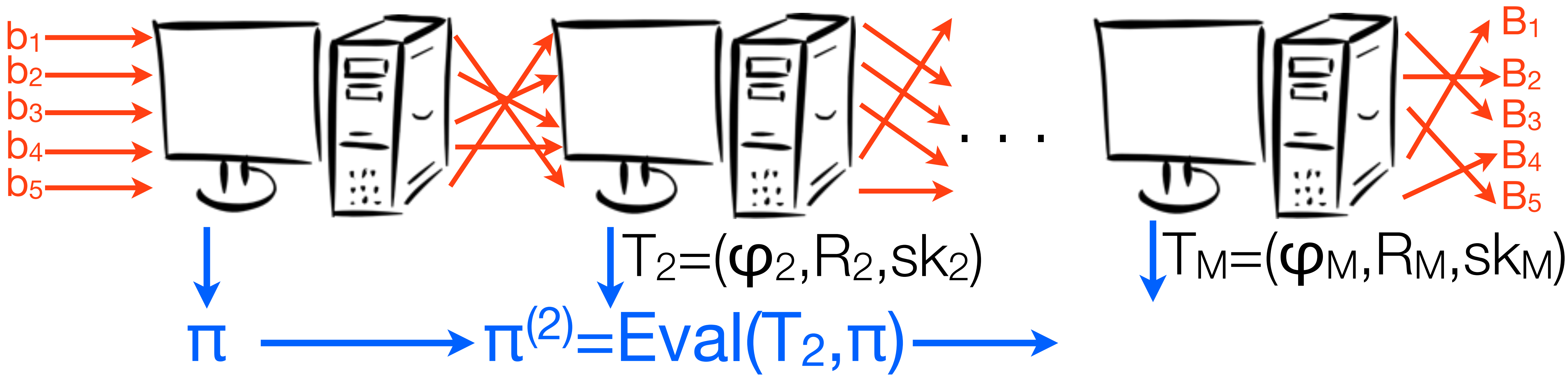

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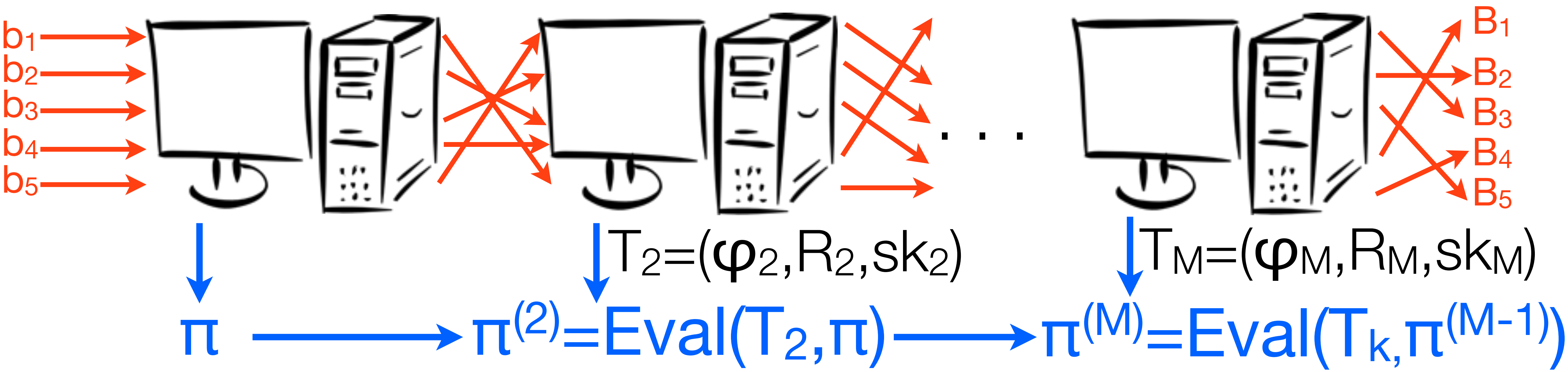

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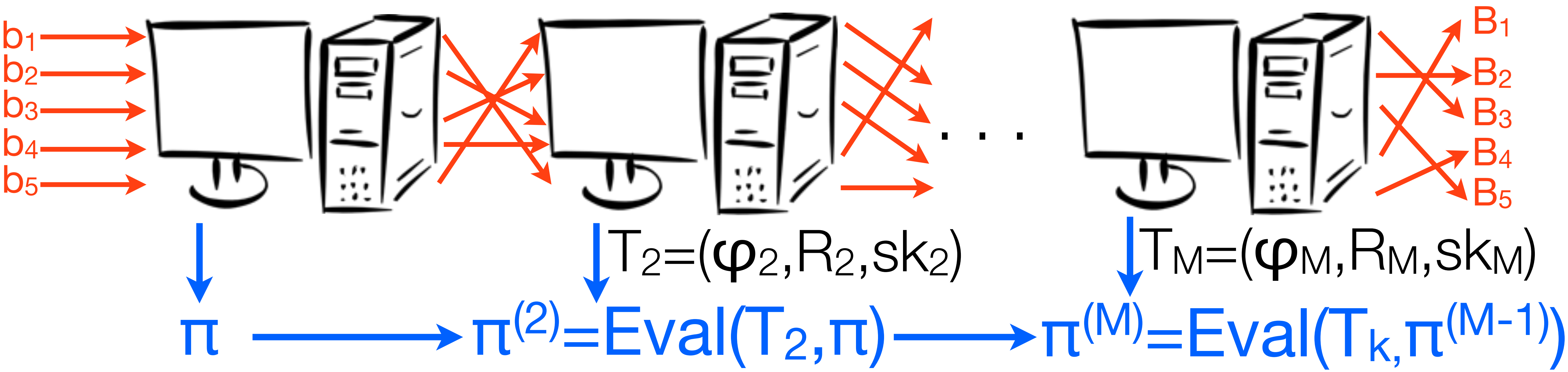

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So if there are L ciphertexts and M servers, proof size can be O(L+M)

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KeyGen

Enc

Compact threshold decryption

C=Enc(pk,m)→ 

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$\rightarrow s_1$

KeyGen
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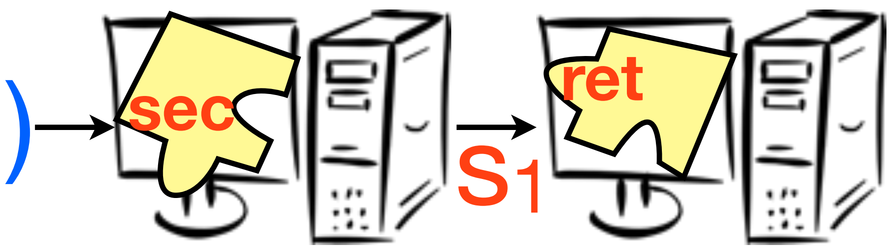

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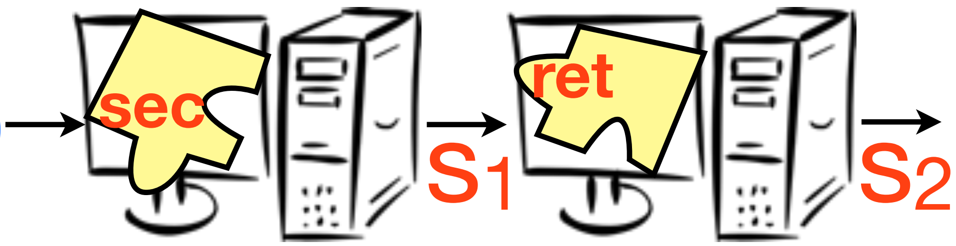

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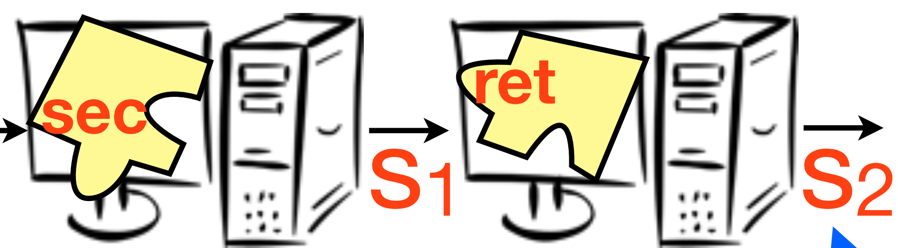

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Shares contain proof of correctness

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ShareDec
(ShareProve)

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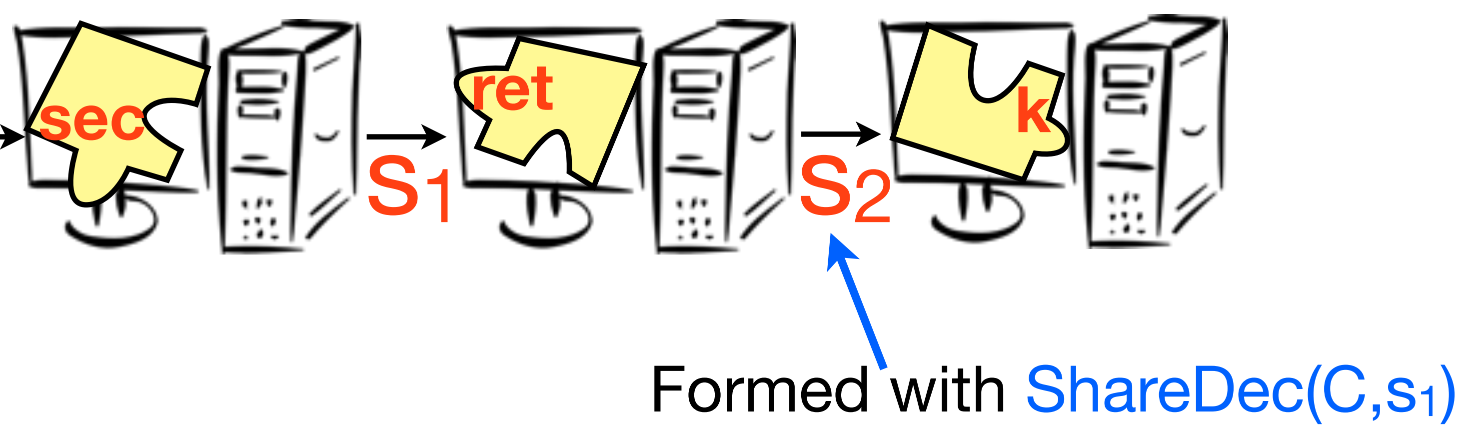

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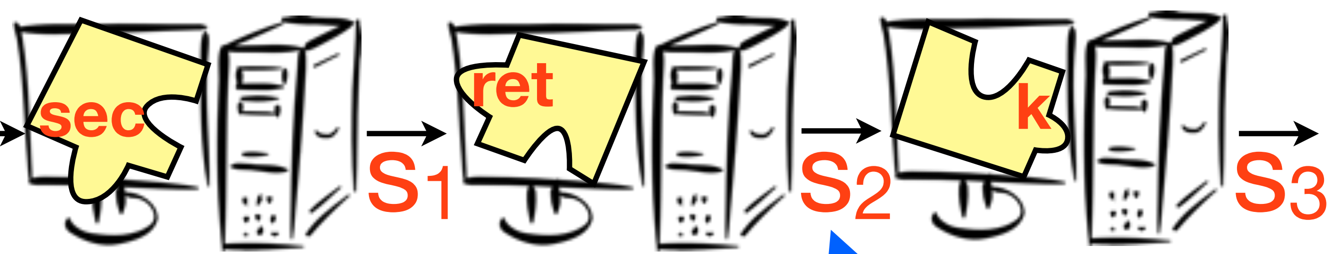

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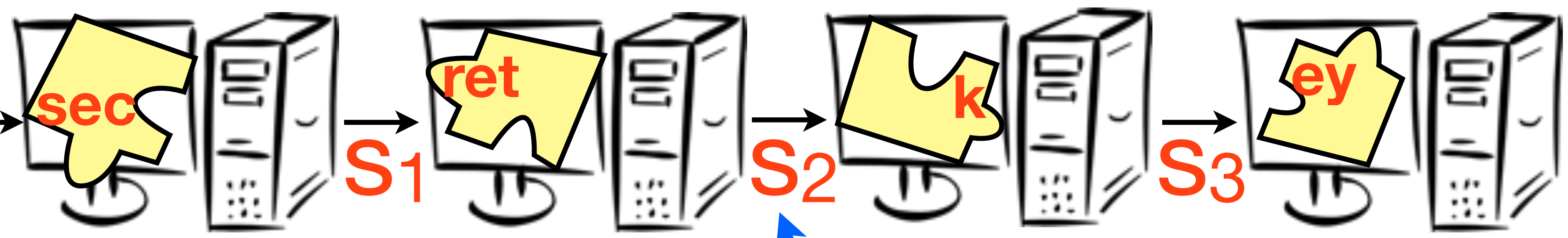

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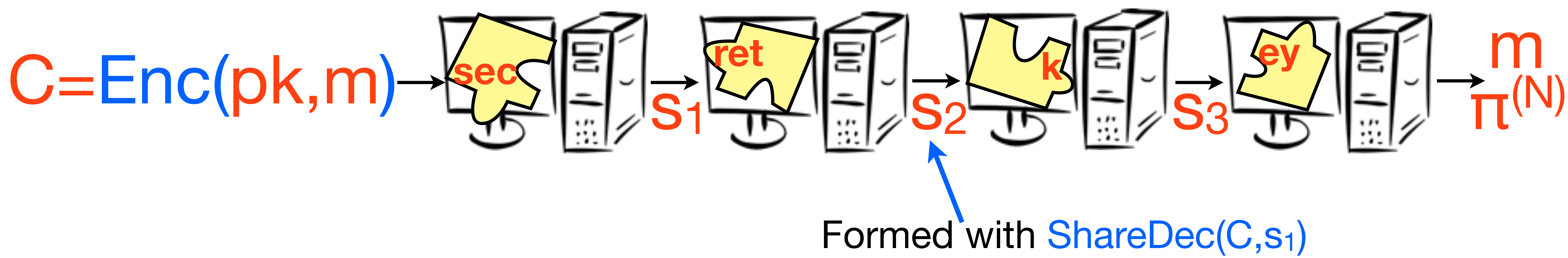

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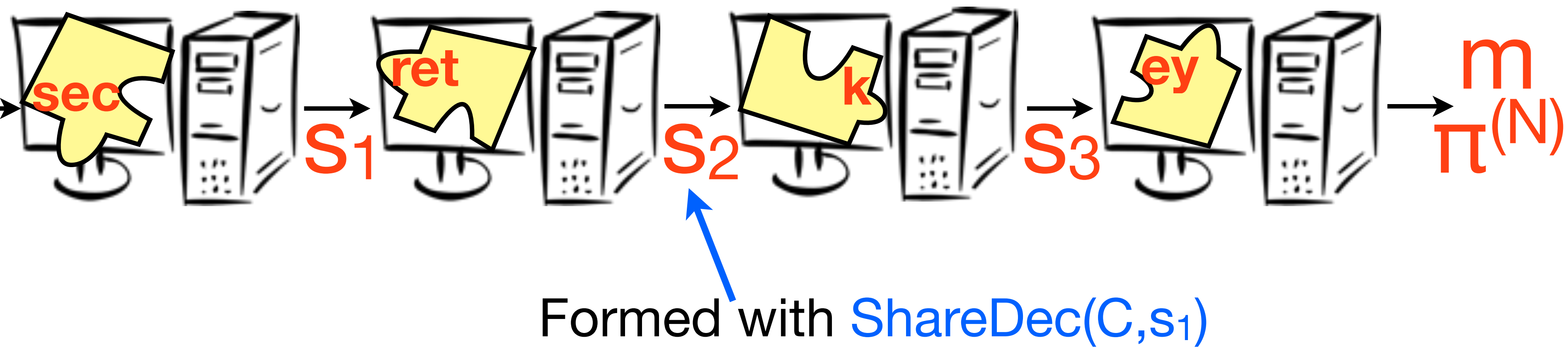

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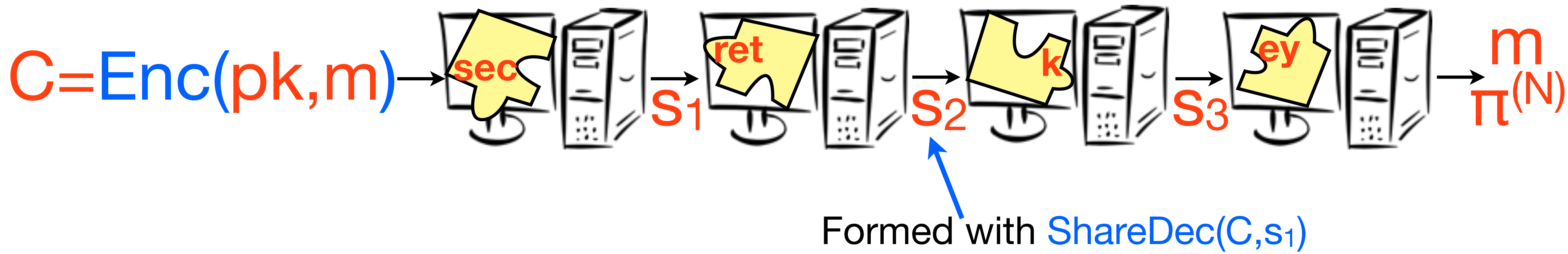

Shares contain proof of correctness

Servers can decrypt in any order; not fixed

Once again, final proof $\pi^{(N)}$ suffices for whole decryption, meaning total proof size can again be O(L+N) instead of O(LN) (again, under an appropriate definition)

KeyGen
Enc
ShareDec
(ShareProve)
ShareVerify

Outline

Definitions

Shuffling and decrypting

A compact verifiable shuffle

Threshold decryption

A voting scheme

Conclusions

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Dec(crs,sk,(u,v,w)): return $u^{-1/\alpha}v^{-1/\beta}w$

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Basically, alter their proofs and make them malleable (i.e., show they satisfy CM-friendliness)

End up with CRS of size O(M), proofs of size O(L+M) (improvement over [CKLM12], which had constant-sized CRS but proofs of size $O(L^2+M)$)

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To split BBS decryption key sk = (α,β), just pick $\alpha_1,\beta_1,\ldots,\alpha_{N-1},\beta_{N-1} \leftarrow F_p$ and set $\alpha_N = -1/\alpha - \sum \alpha_i$ and $\beta_N = -1/\beta - \sum \beta_i$; then $\alpha_1 + \ldots + \alpha_N = -1/\alpha$ and $\beta_1 + \ldots + \beta_N = -1/\beta$

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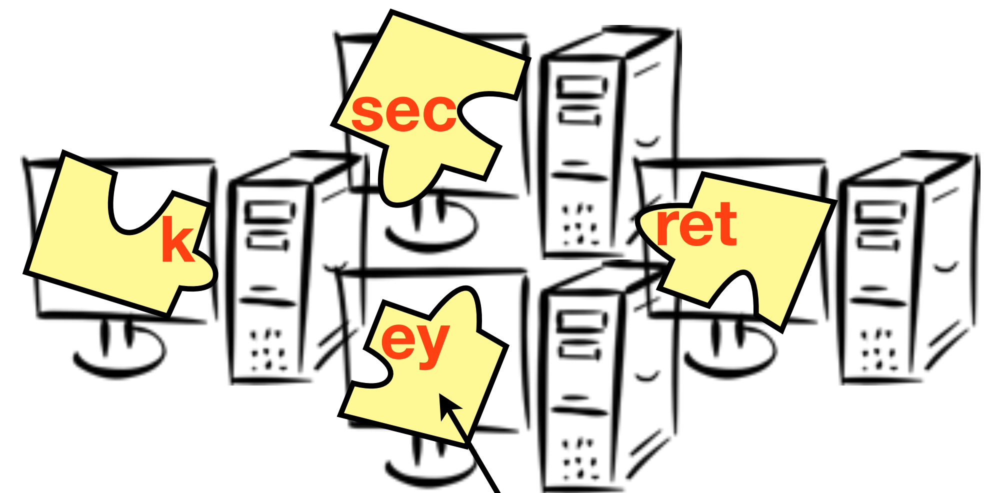

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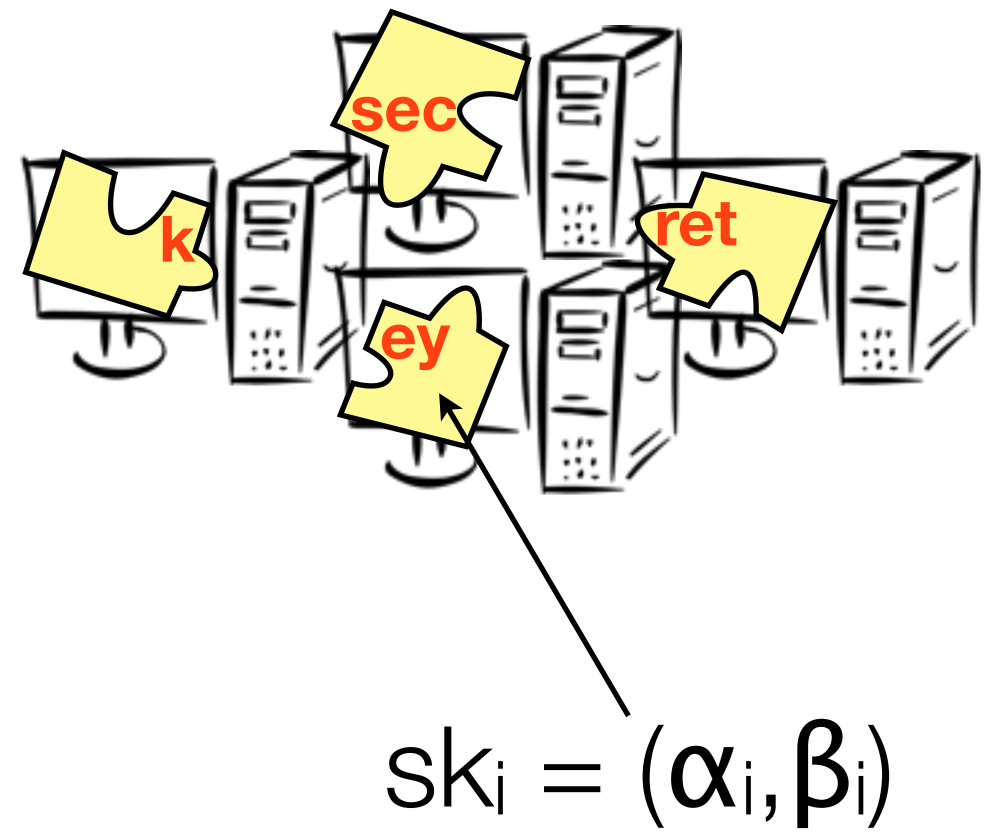

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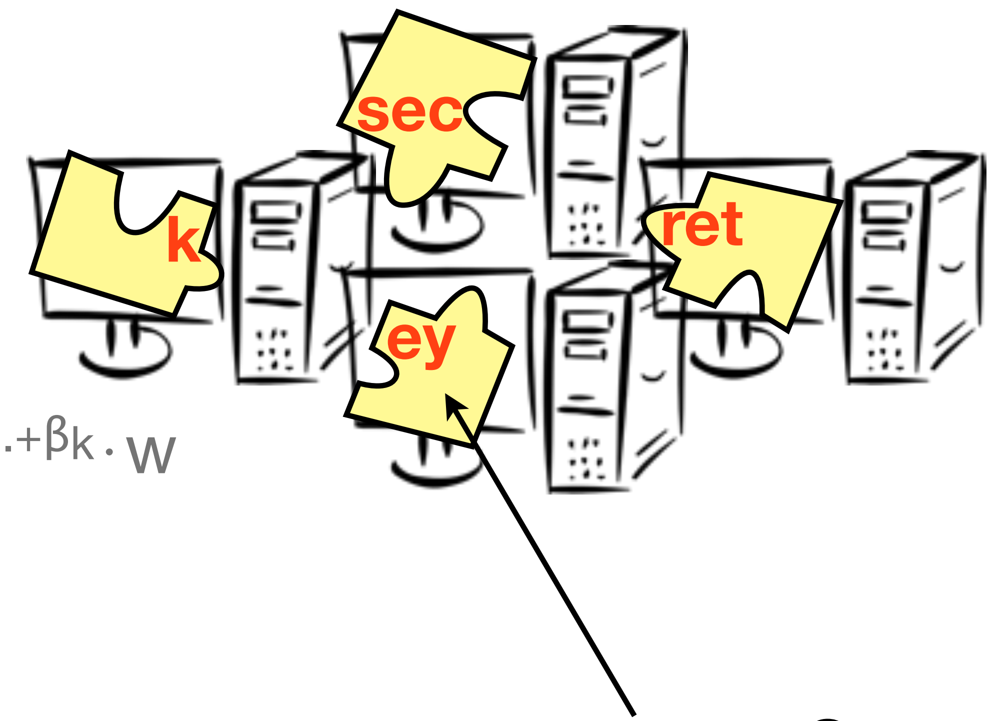

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Also want verification key $vk = (\mathrm{Com}(sk_1)=(\mathrm{Com}(\alpha_1),\mathrm{Com}(\beta_1)),\ldots,\mathrm{Com}(sk_N))$

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partial decryption → s; participants → I; proof of correct partial decryption → π

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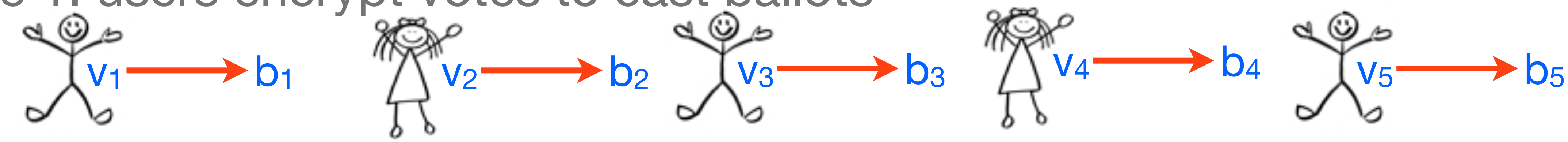
Instantiating cryptographic voting

Phase 1: users encrypt votes to cast ballots



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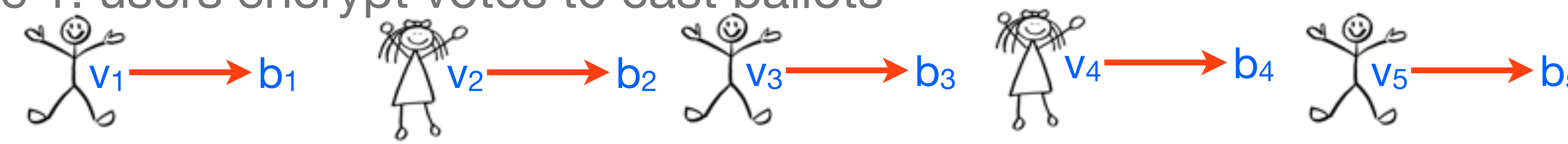

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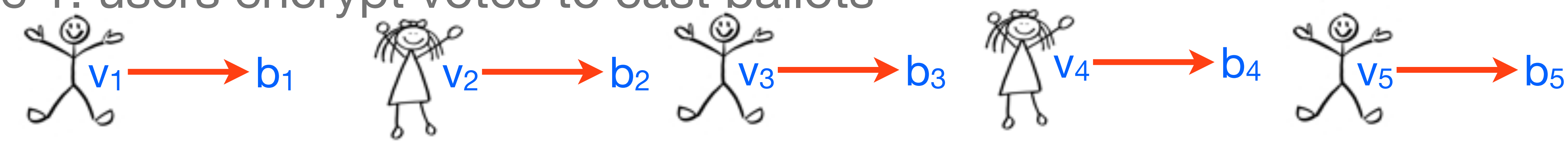

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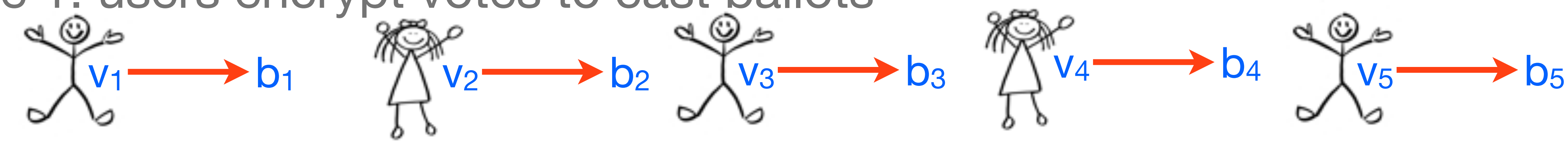

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The vk for threshold decryption is size O(N); for shuffles the crs is size O(M)

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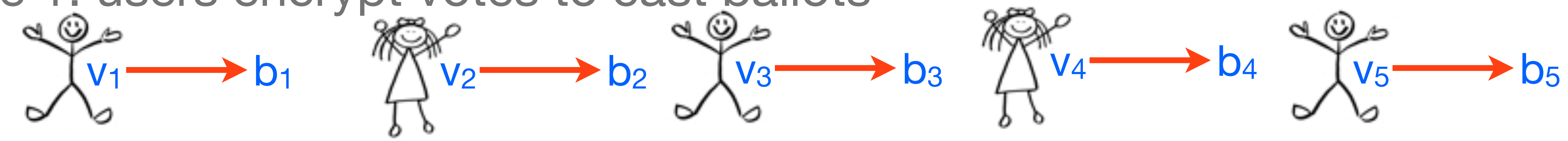

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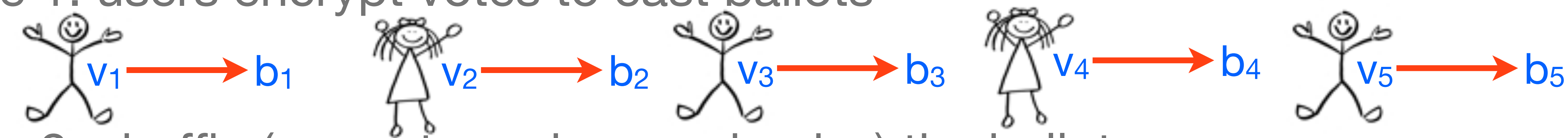

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Instantiating cryptographic voting

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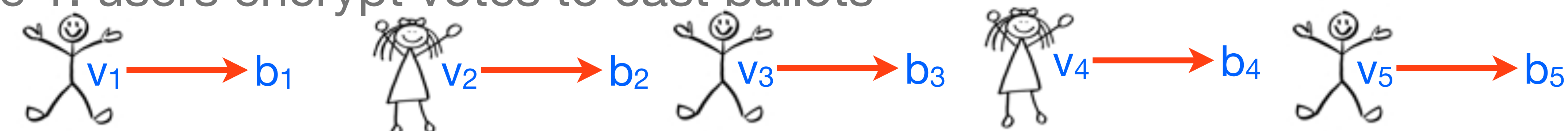

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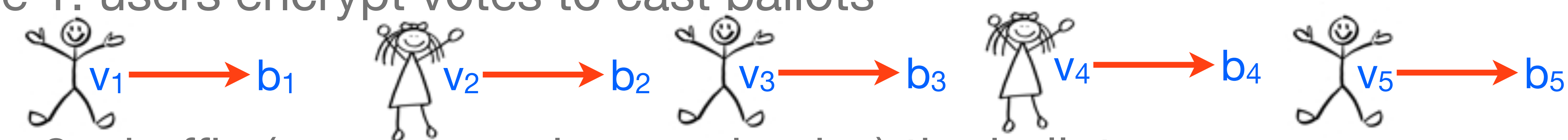

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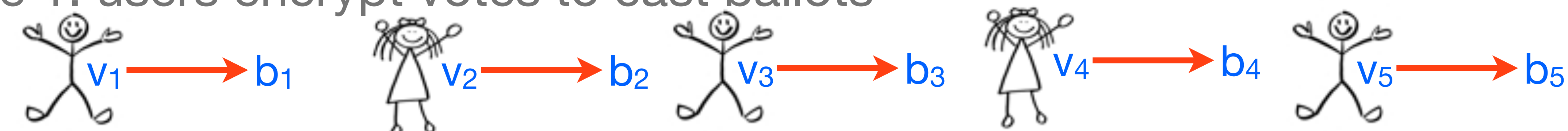

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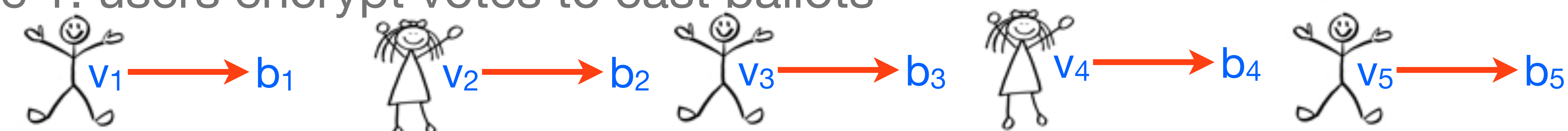

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Phase 3: threshold decrypt the shuffled ballots

Instantiating cryptographic voting

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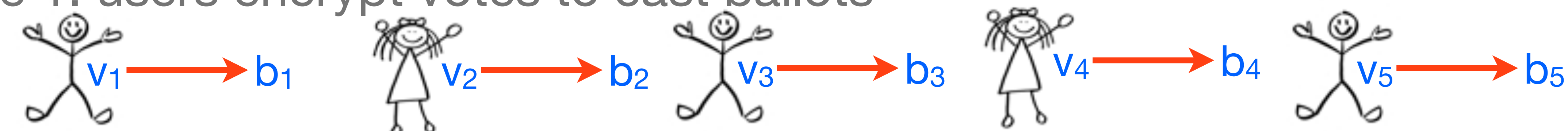

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Phase 3: threshold decrypt the shuffled ballots

Resulting proof from cumulative threshold decryption is O(L+N), so total verifier input size? $O(M) + O(N) + O(L+M) + O(L+N) = O(L+M+N)$

Instantiating cryptographic voting

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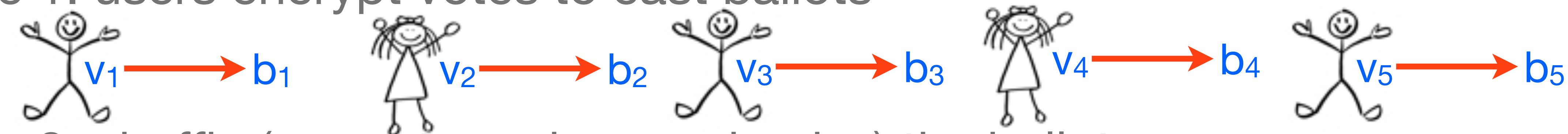

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Also show this satisfies notion of vote privacy for elections

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Thanks!
Any questions?

Regular verifiable threshold decryption [SG98]

Regular verifiable threshold decryption [SG98]

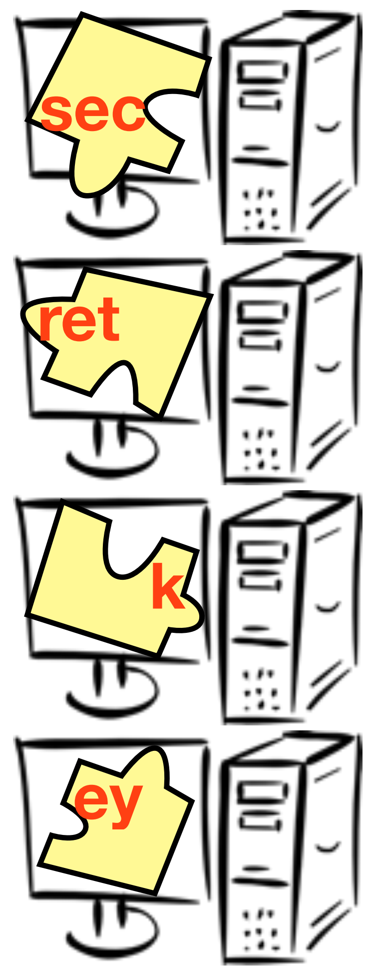
$C = Enc(pk, m)$

Regular verifiable threshold decryption [SG98]

C=Enc(pk,m)

Regular verifiable threshold decryption [SG98]

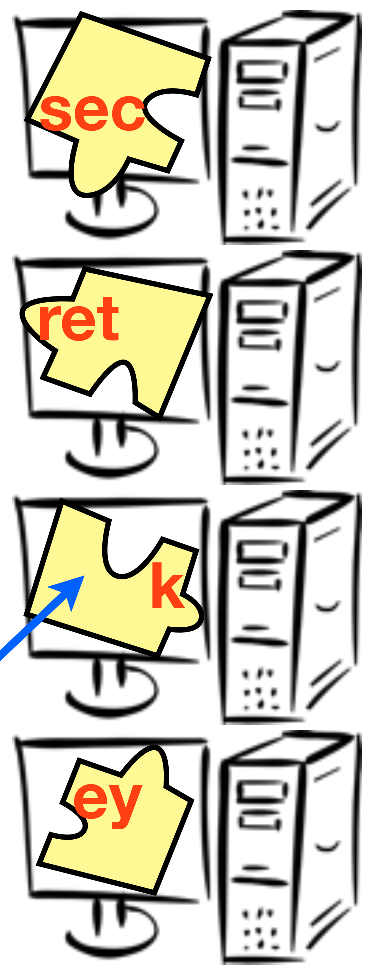
C=Enc(pk,m)



KeyGen

Regular verifiable threshold decryption [SG98]

C=Enc(pk,m)



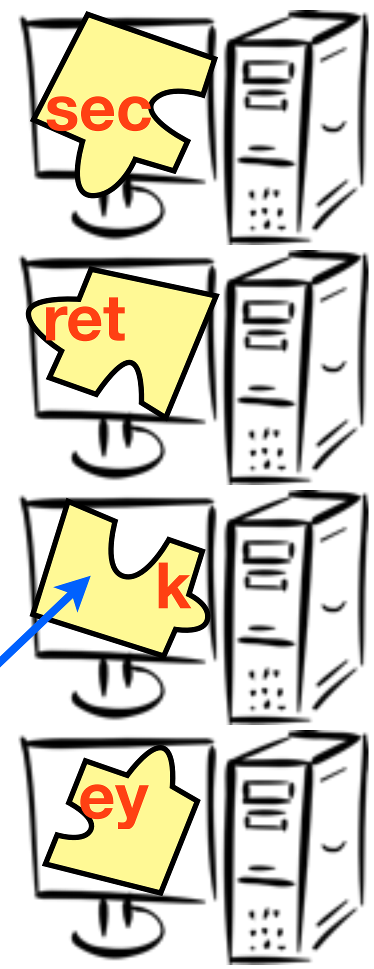
Formed with KeyGen

KeyGen

Regular verifiable threshold decryption [SG98]

C=Enc(pk,m)

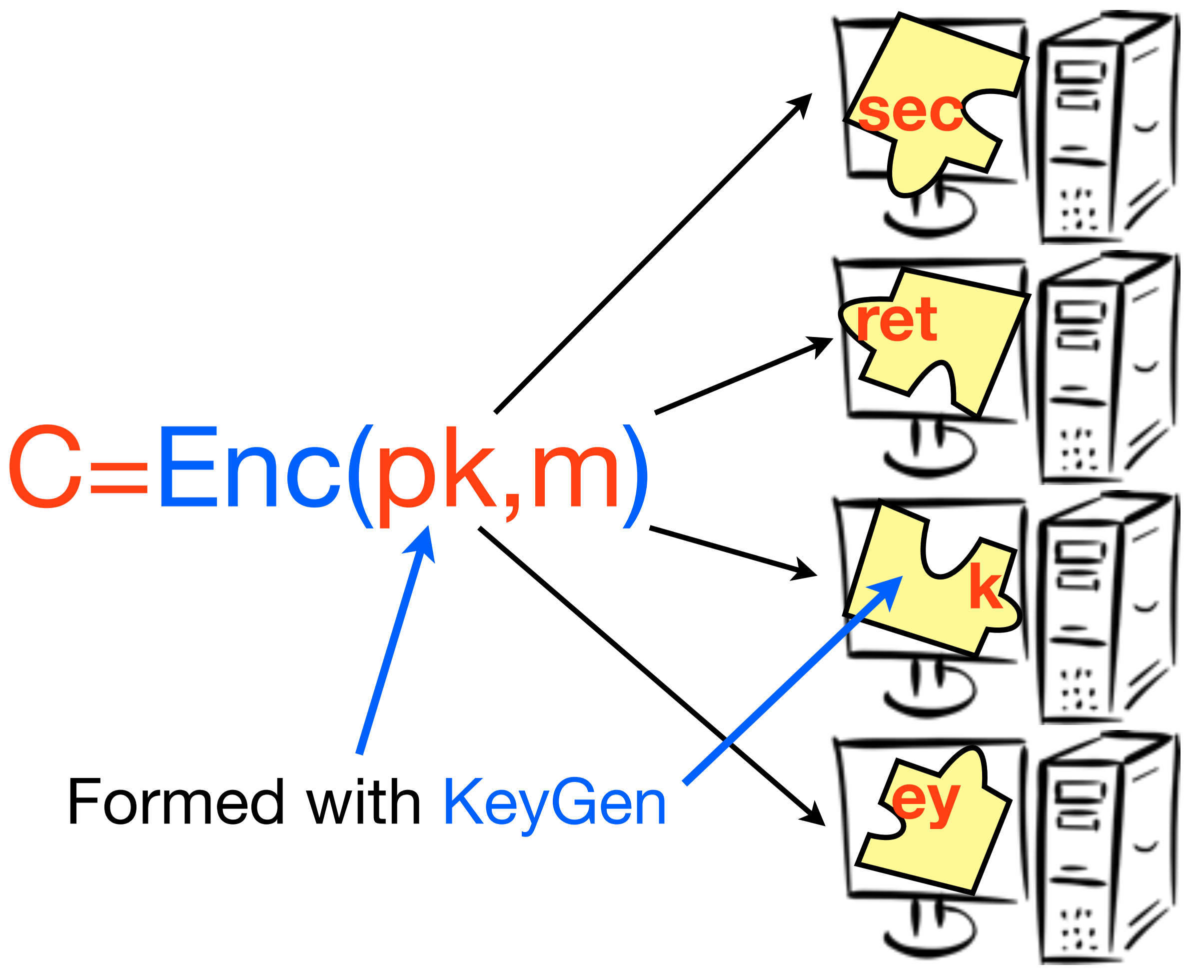
Formed with KeyGen



KeyGen

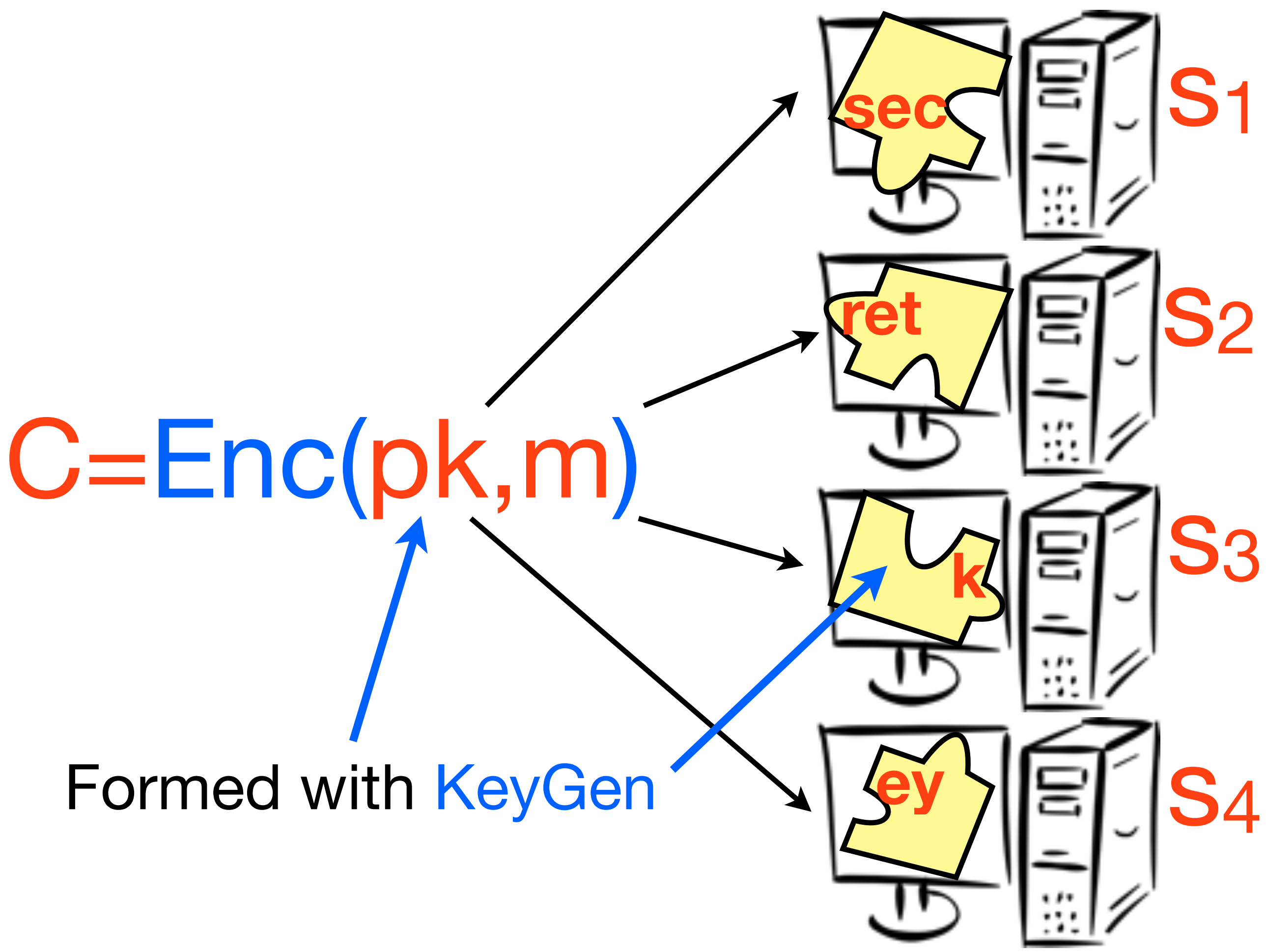
Enc

Regular verifiable threshold decryption [SG98]



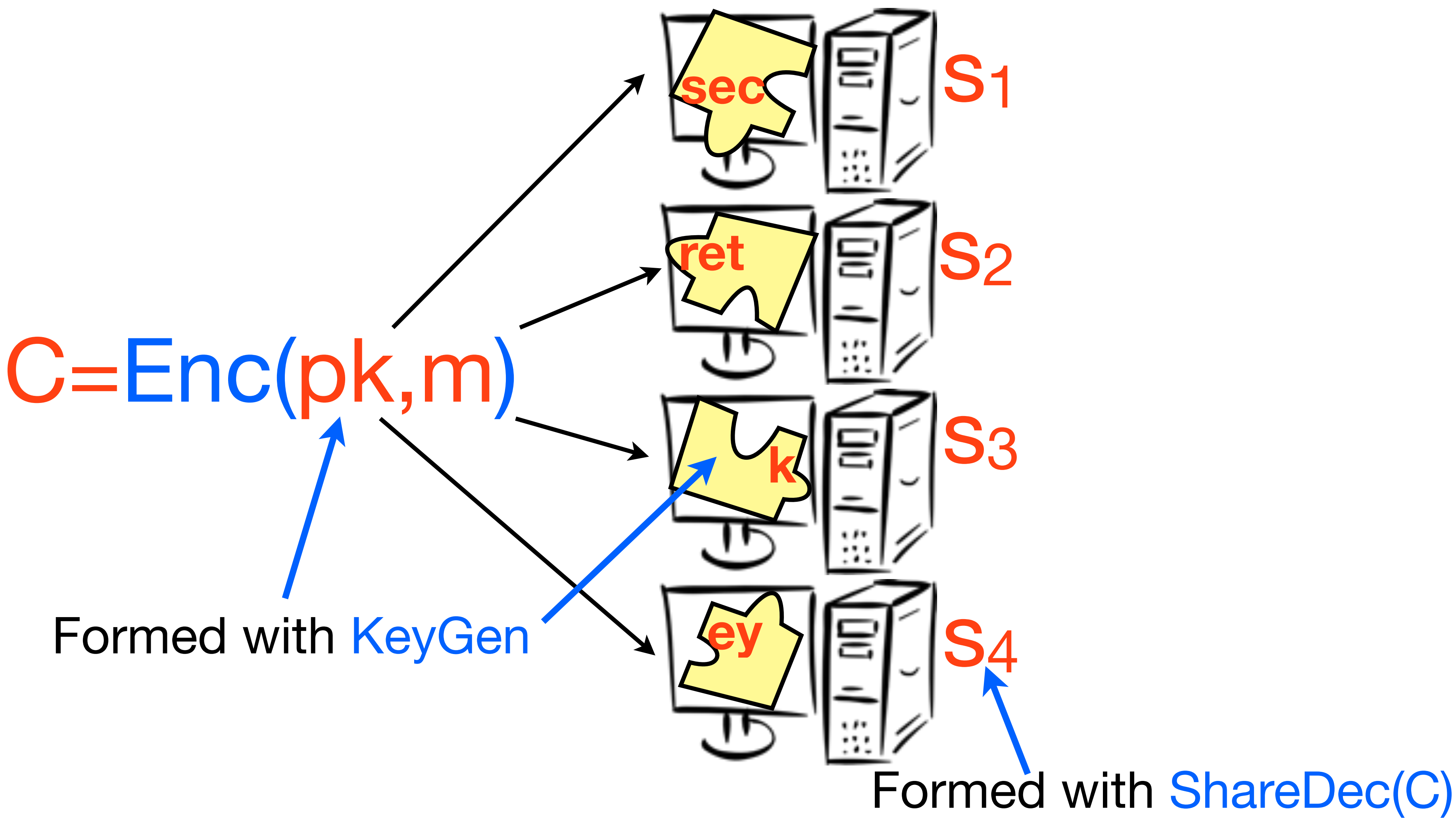
KeyGen
Enc

Regular verifiable threshold decryption [SG98]



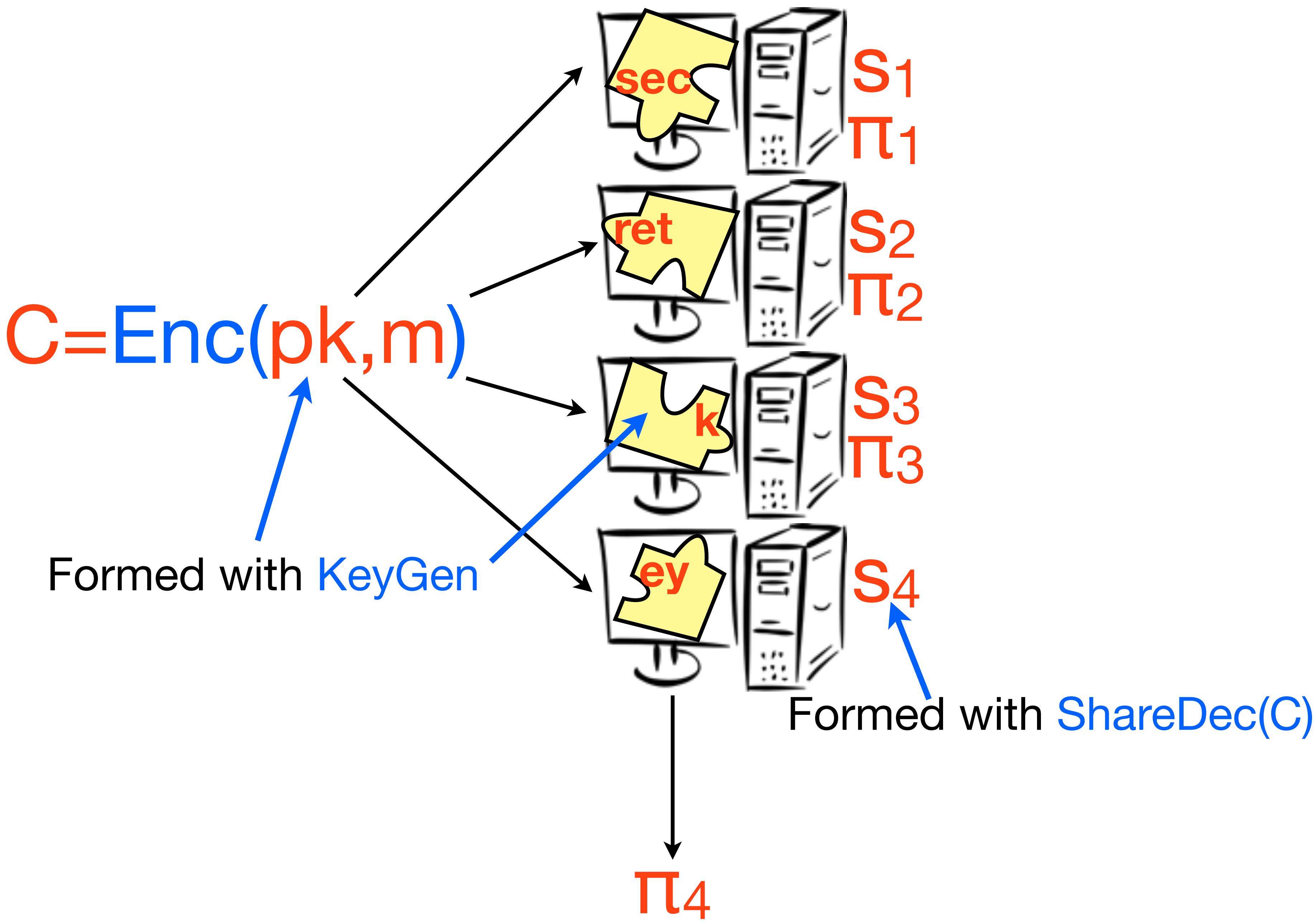
KeyGen
Enc

Regular verifiable threshold decryption [SG98]



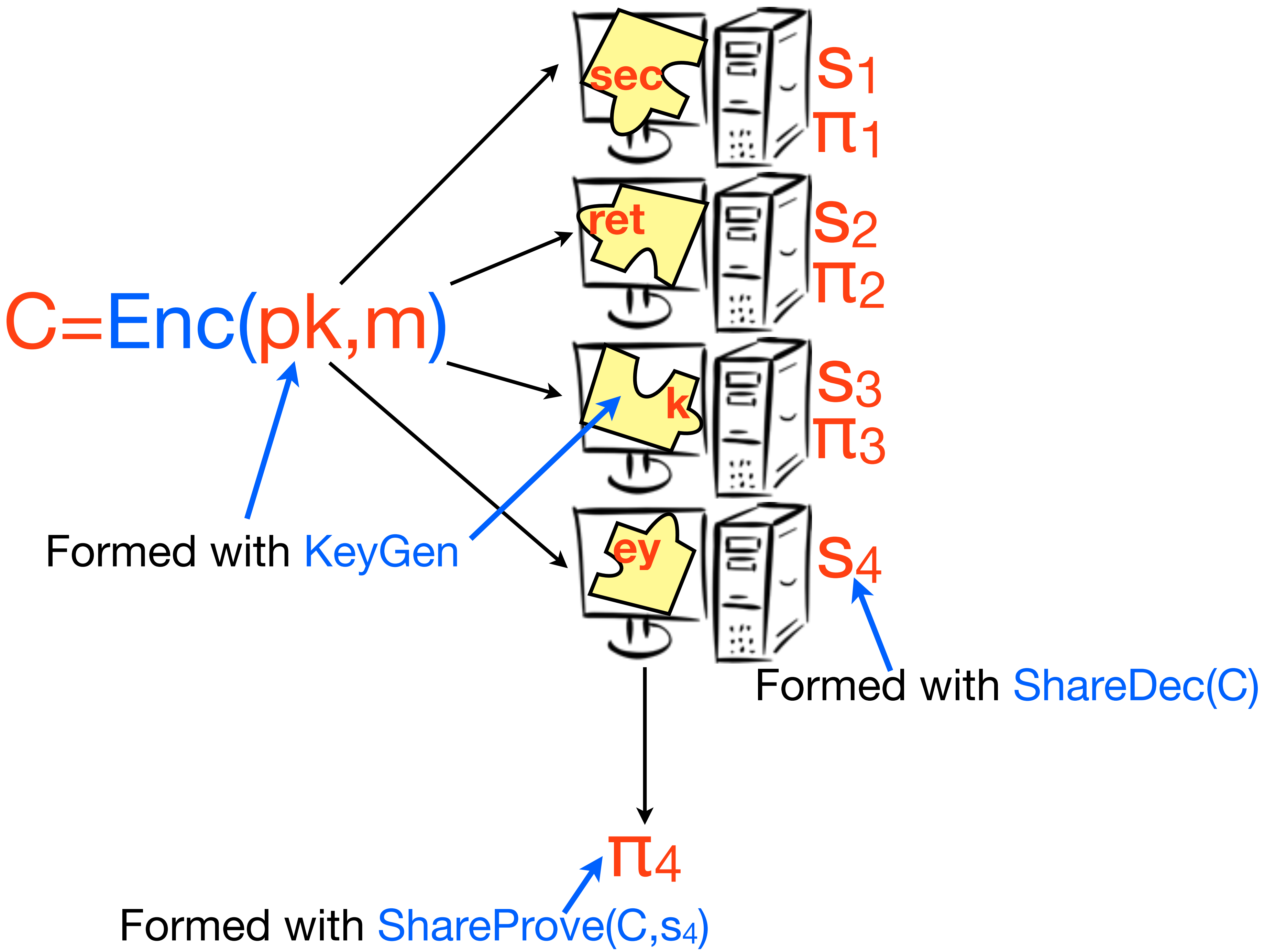
KeyGen
Enc
ShareDec

Regular verifiable threshold decryption [SG98]



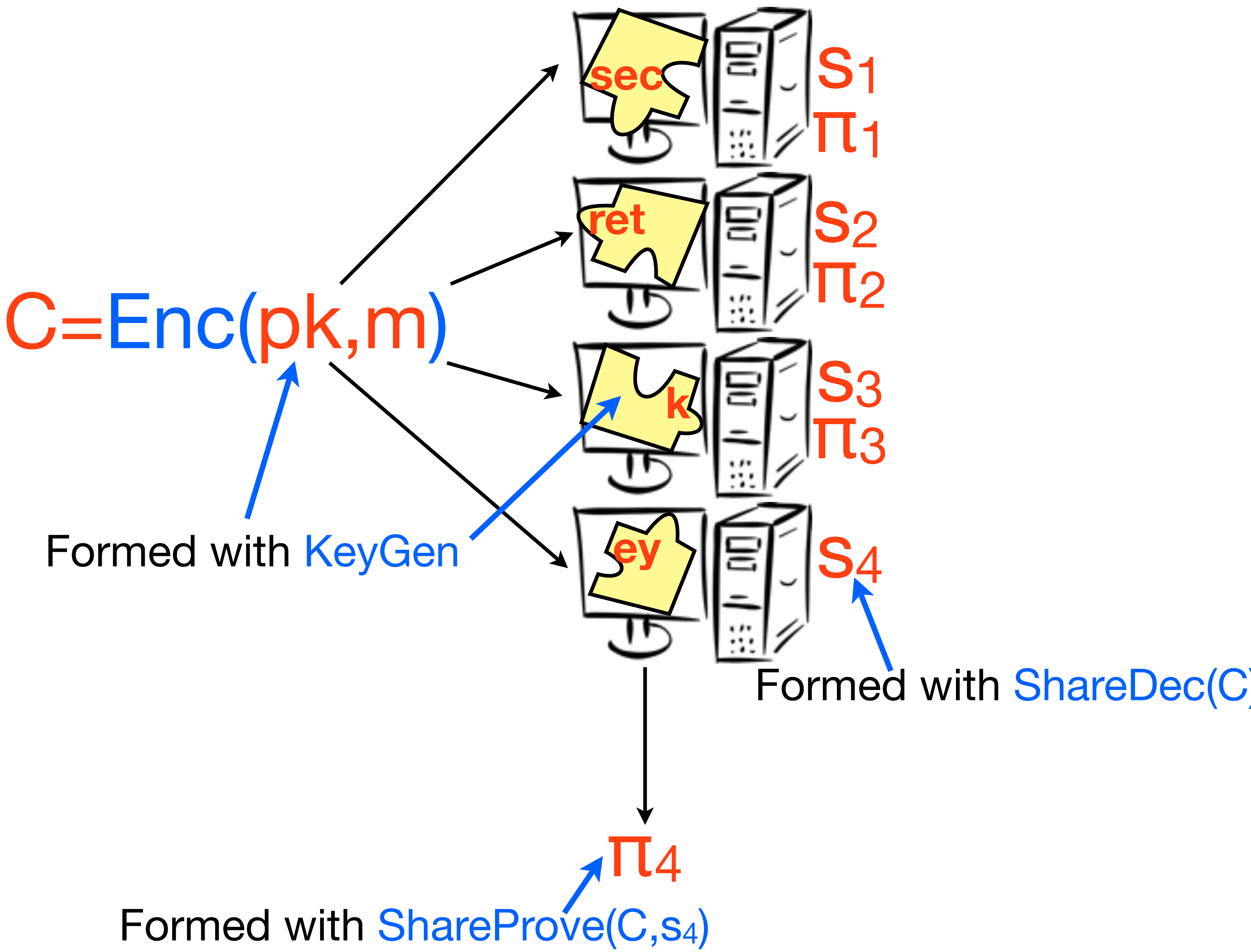
KeyGen
Enc
ShareDec

Regular verifiable threshold decryption [SG98]



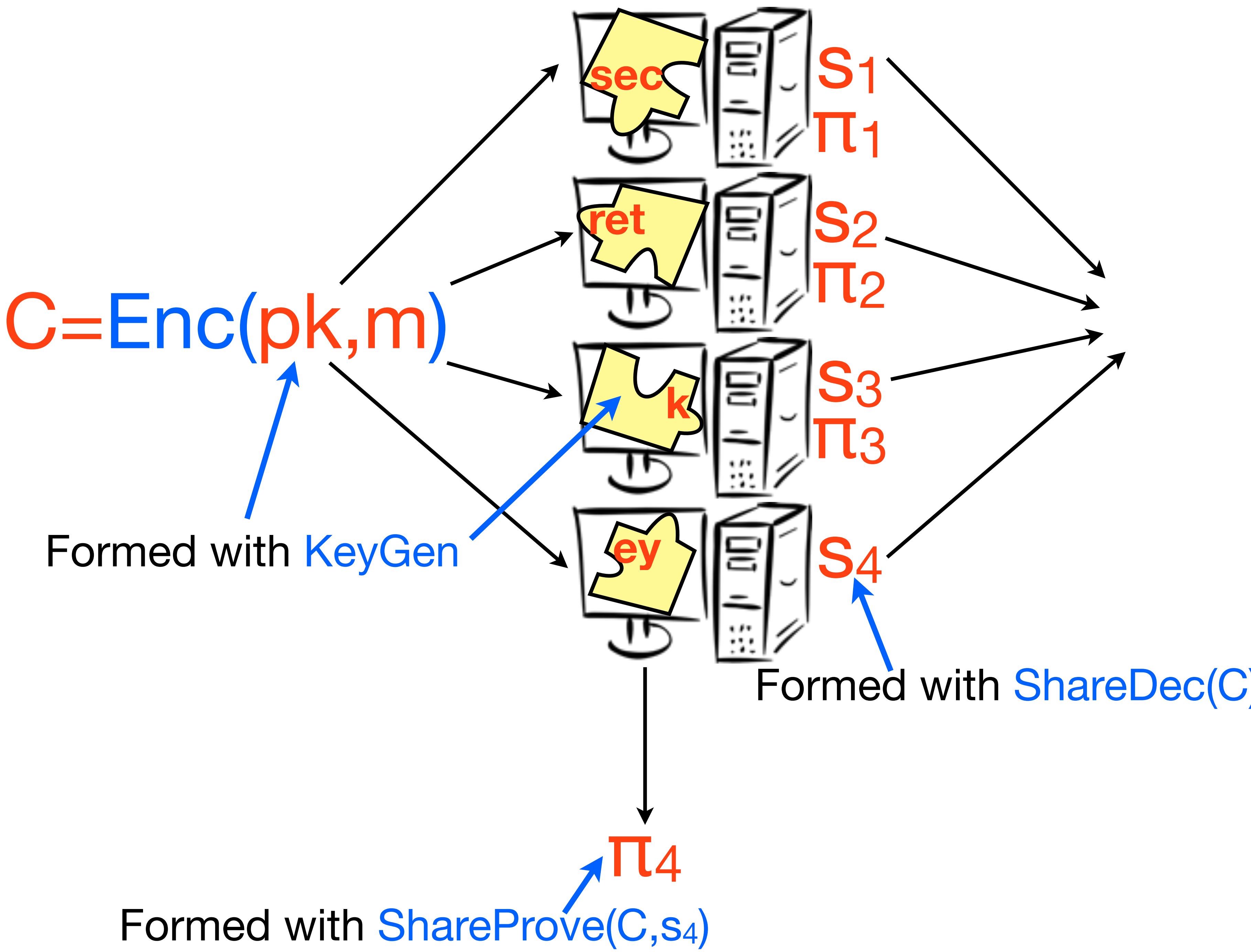
KeyGen
Enc
ShareDec
ShareProve

Regular verifiable threshold decryption [SG98]



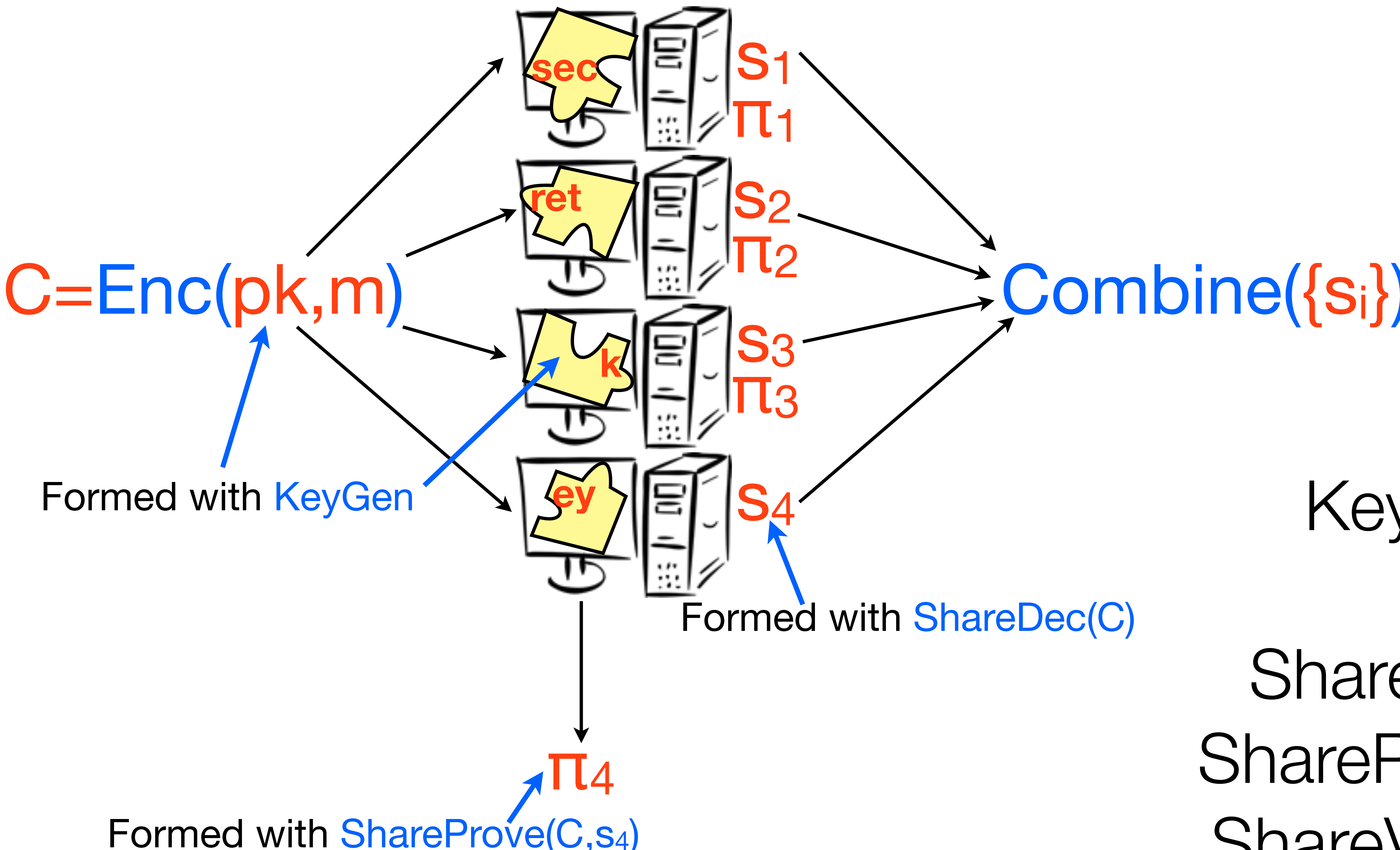
KeyGen
Enc
ShareDec
ShareProve
ShareVerify

Regular verifiable threshold decryption [SG98]



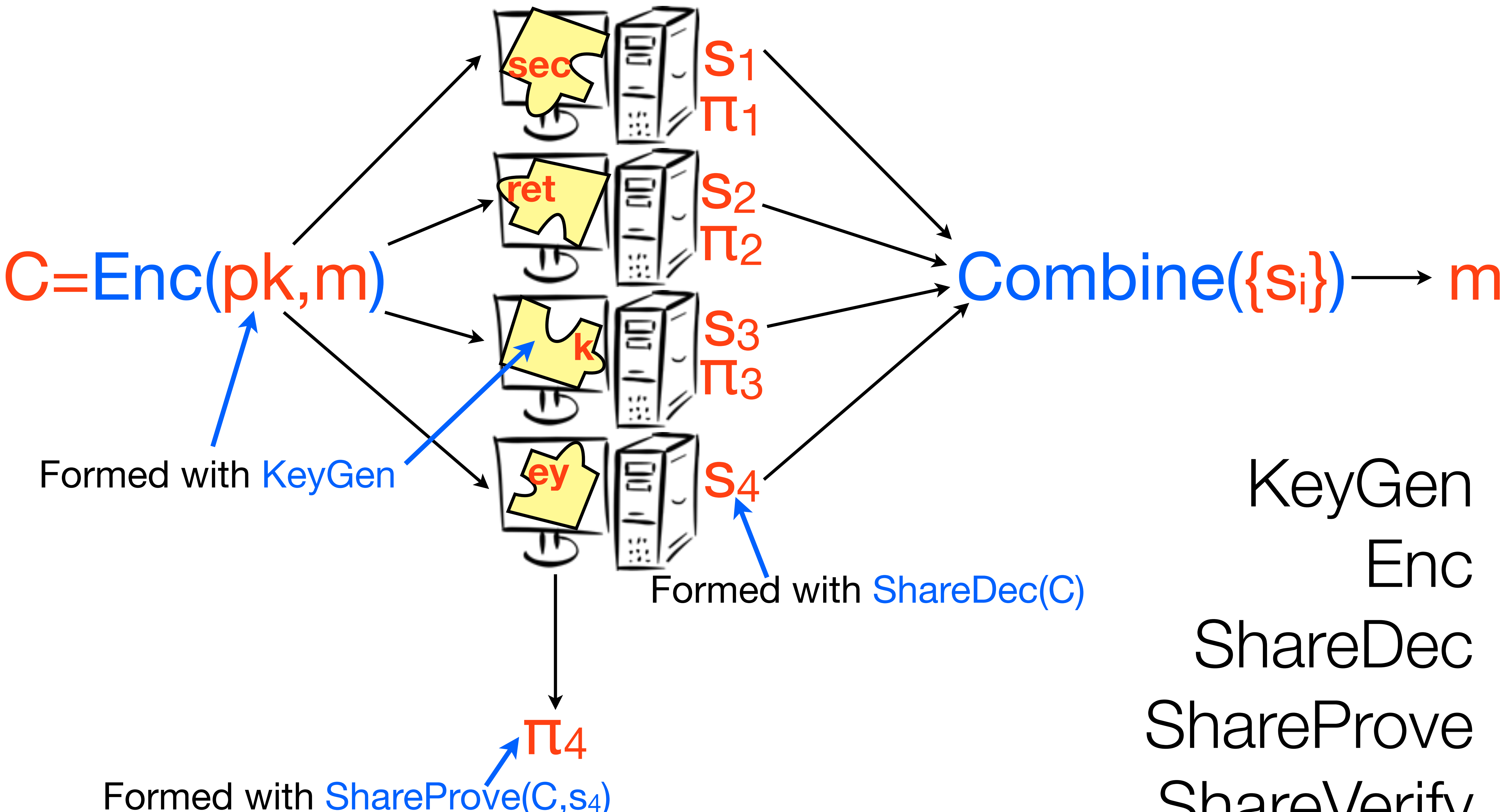
KeyGen
Enc
ShareDec
ShareProve
ShareVerify

Regular verifiable threshold decryption [SG98]



KeyGen
Enc
ShareDec
ShareProve
ShareVerify
Combine

Regular verifiable threshold decryption [SG98]



KeyGen
Enc
ShareDec
ShareProve
ShareVerify
Combine