

# Succinct Malleable NIZKs and an Application to Compact Shuffles

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Melissa Chase (MSR Redmond)

Markulf Kohlweiss (MSR Cambridge)

Anna Lysyanskaya (Brown University)

**Sarah Meiklejohn (UC San Diego)**

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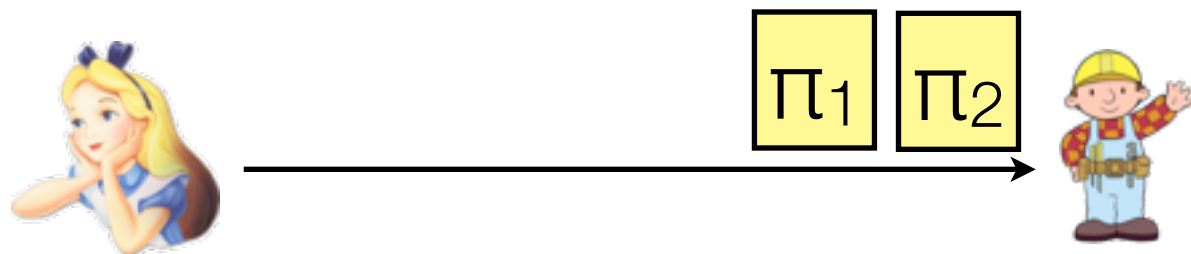
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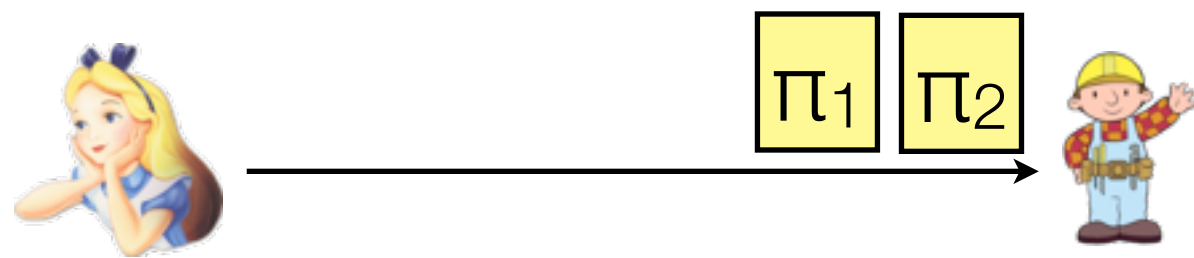
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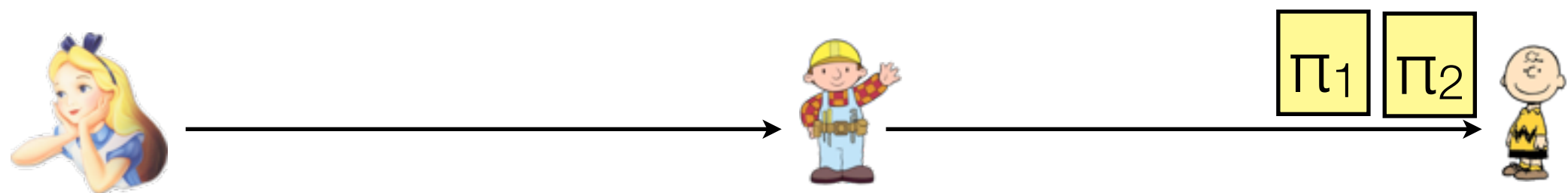


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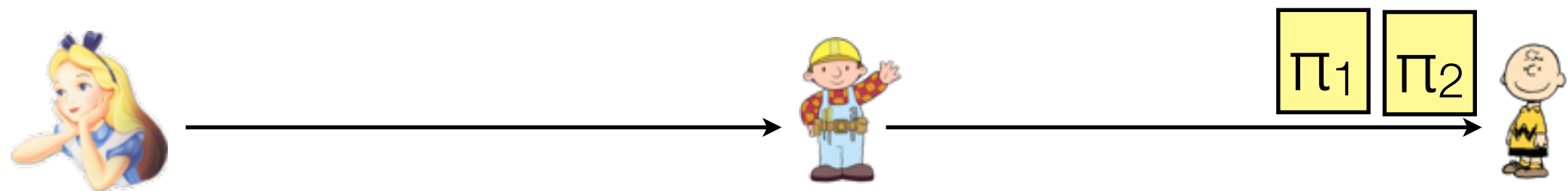
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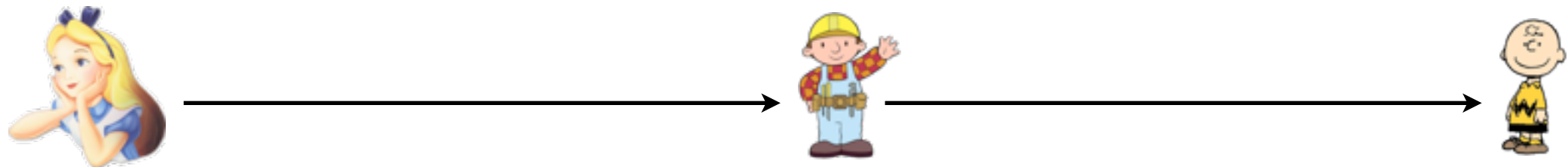
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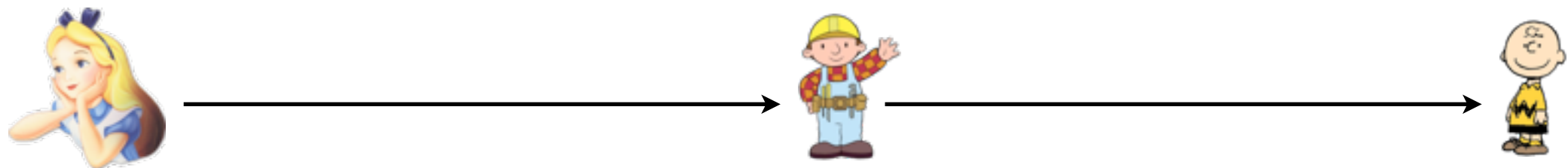
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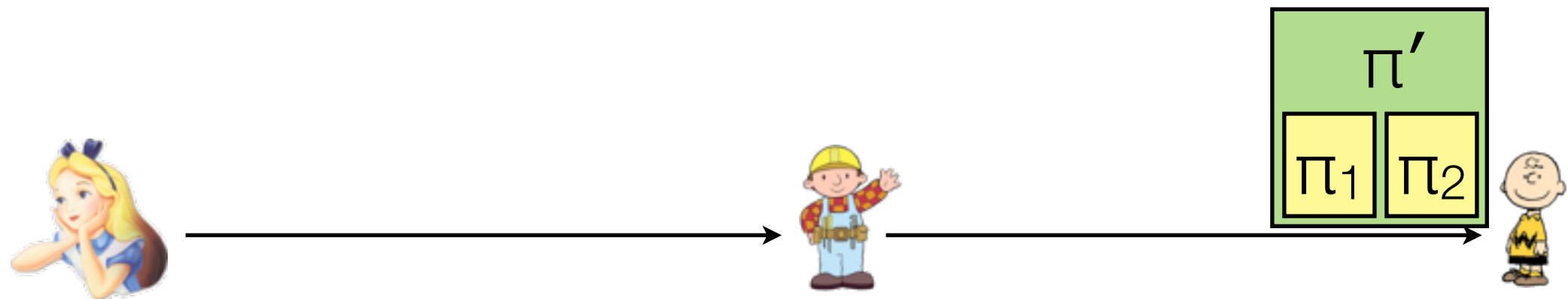
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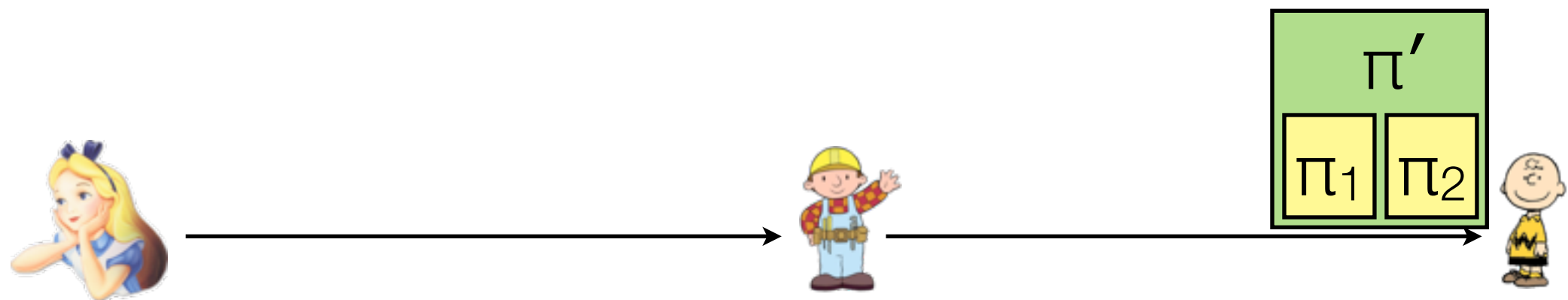
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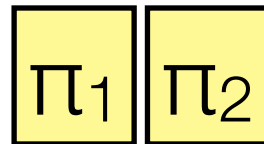
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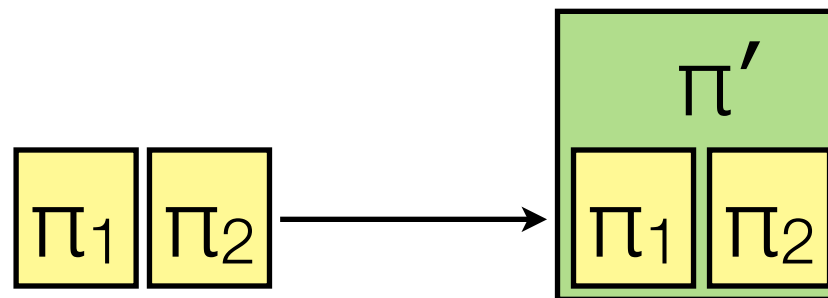




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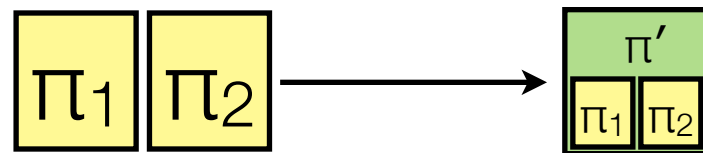
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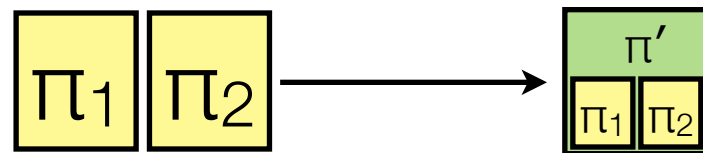
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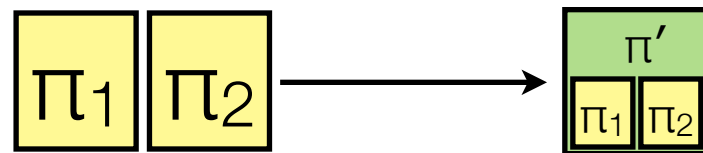


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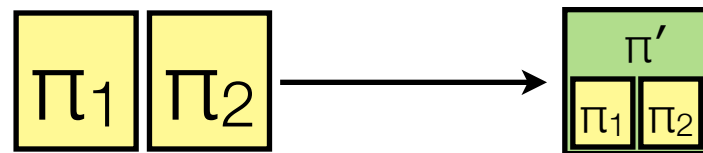
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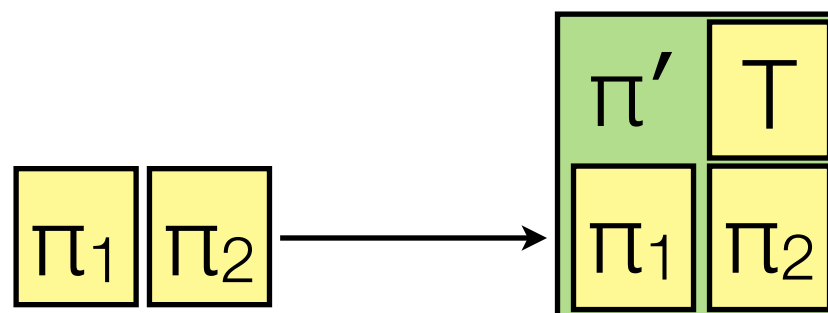
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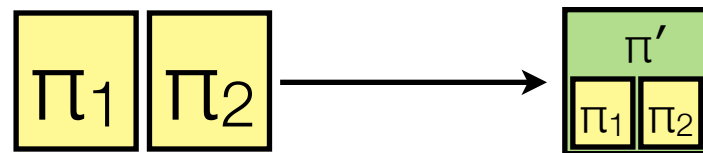
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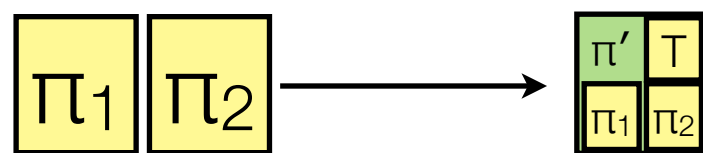
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This would potentially allow for more applications (e.g., **CM-CCA encryption**)

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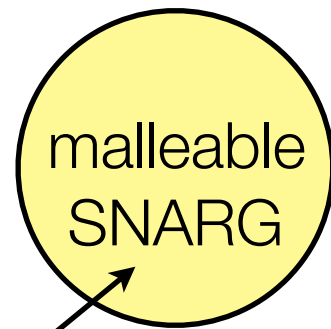
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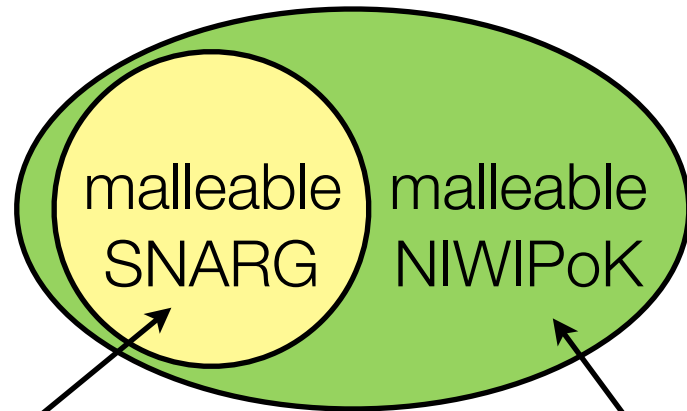


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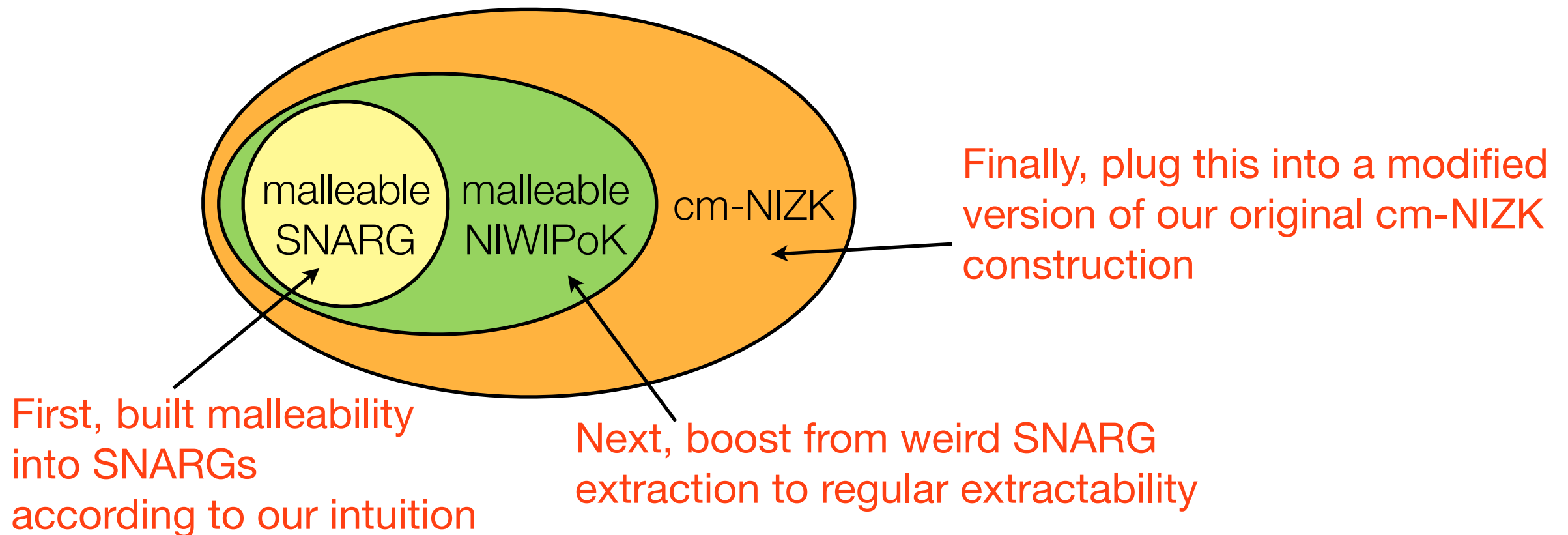
Next, boost from weird SNARG extraction to regular extractability



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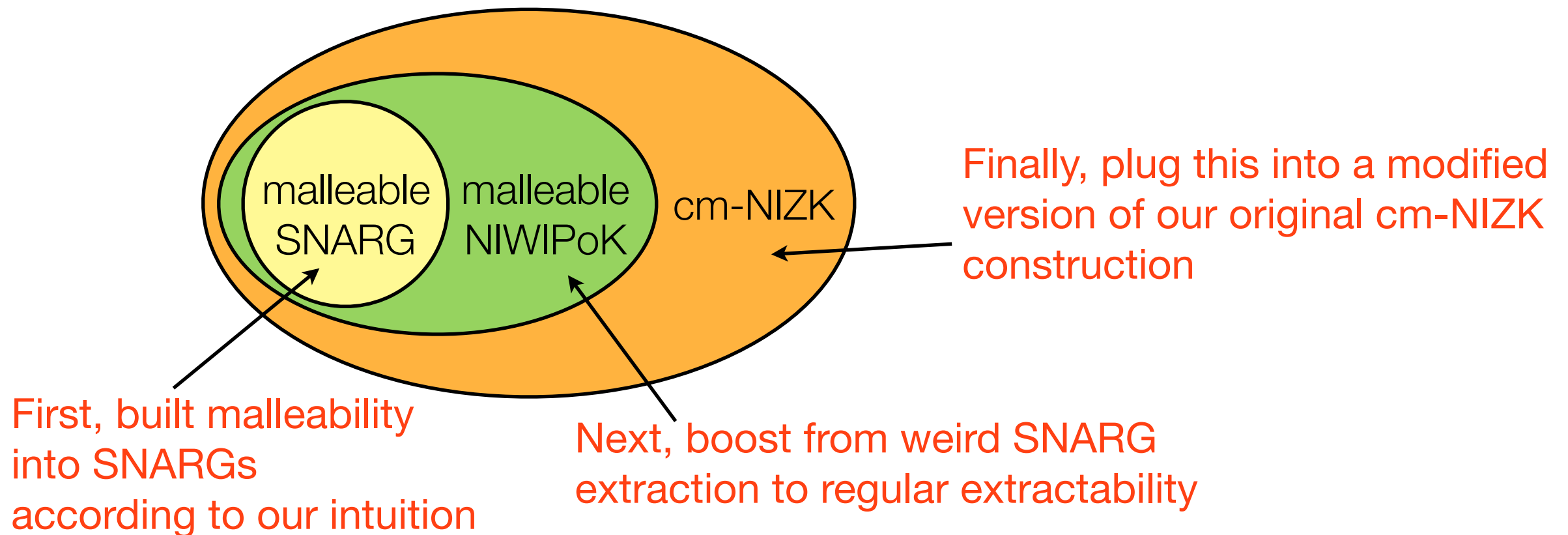
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The end result? A **fully generic cm-NIZK** with a much wider range of malleability (all **t-tiered transformations**) than previously supported, that is **easier** to “plug in” to applications

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(hides fresh vs. transformed)

If a proof is zero knowledge, CM-SSE, and strongly derivation private, then we call it a **cm-NIZK**

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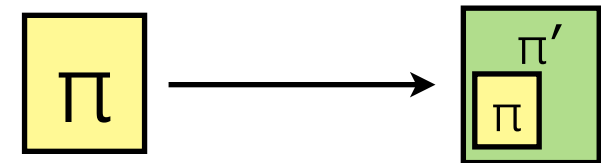
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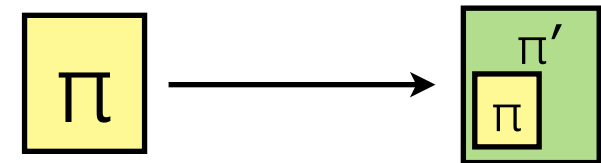
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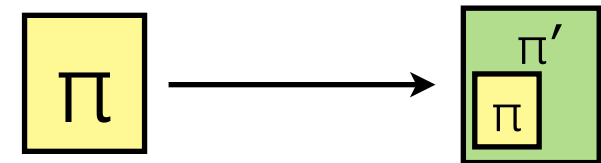
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Constructions of these do exist [AF07,Groth10,...,BCCT12,GGPR13]

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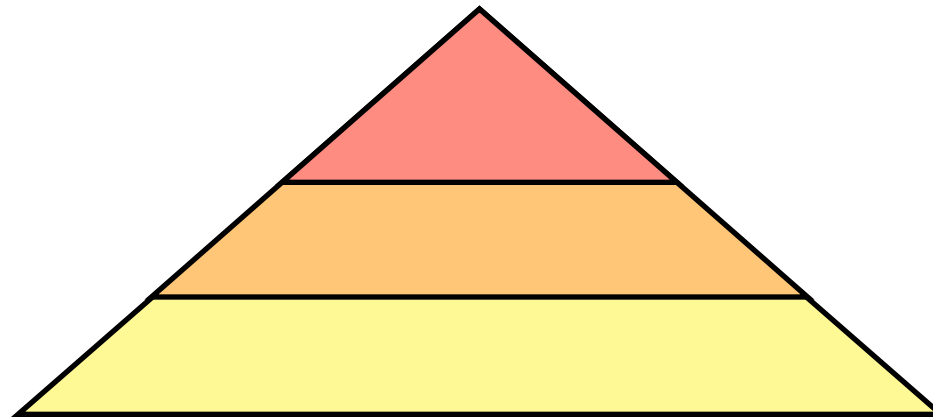
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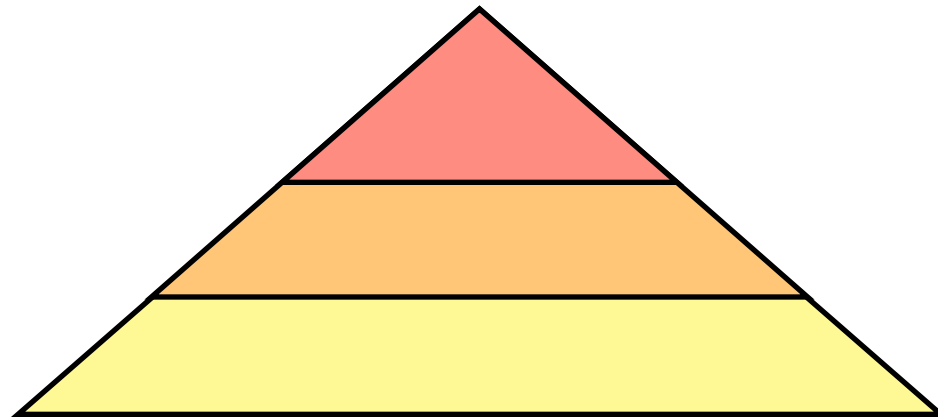


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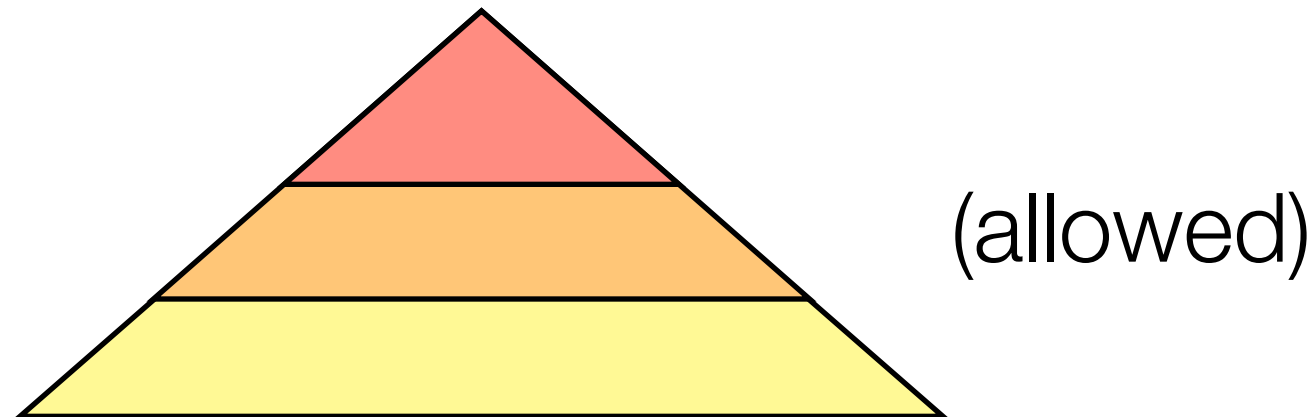
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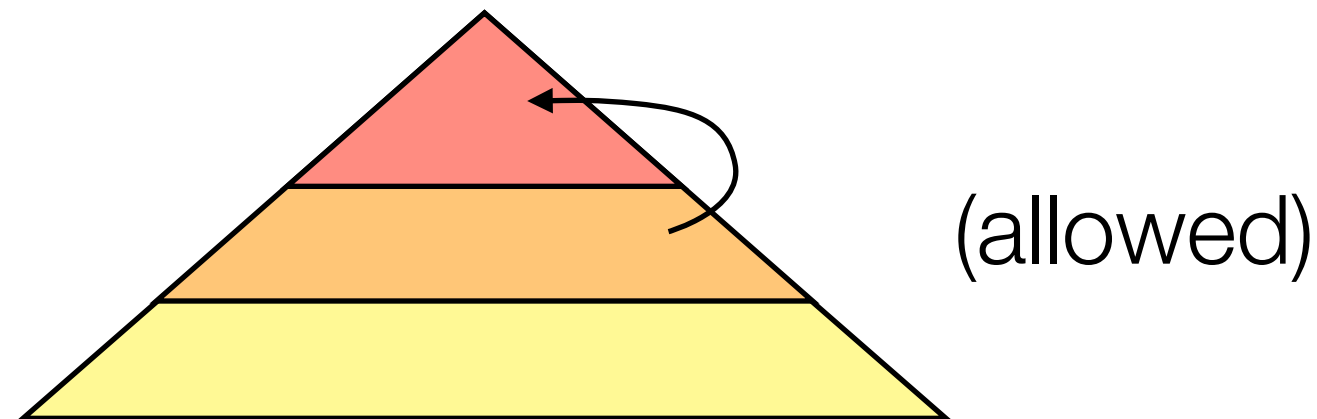
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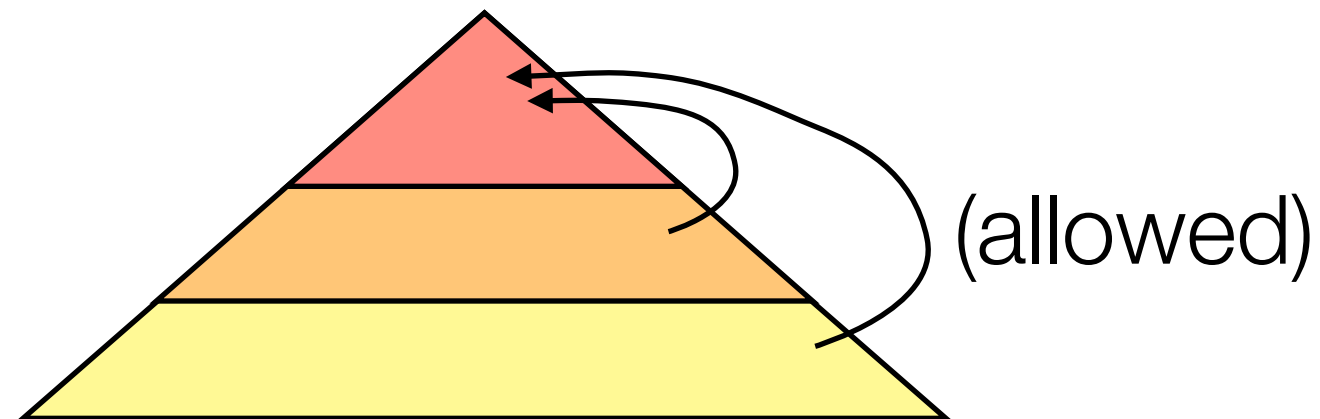
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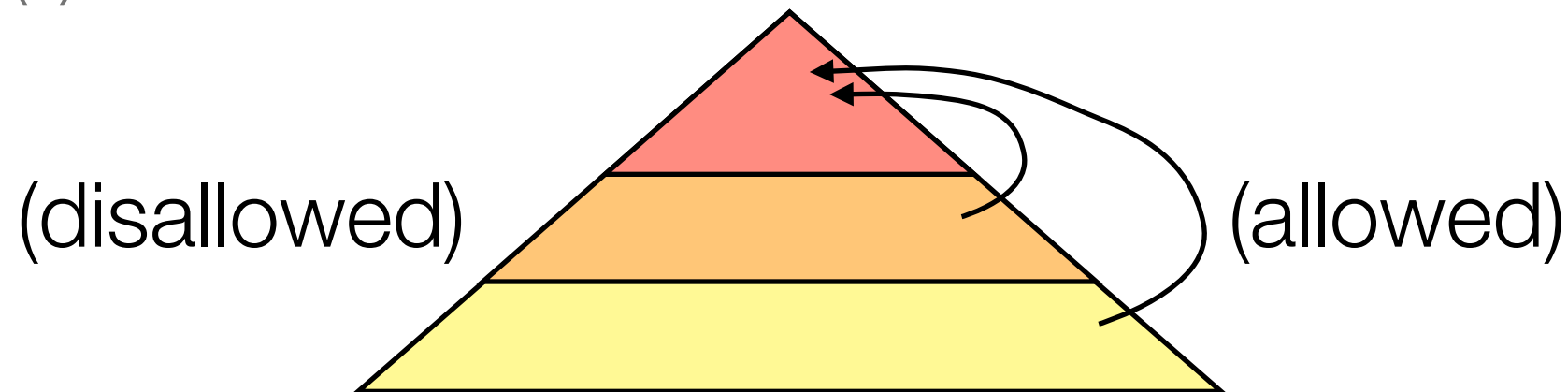
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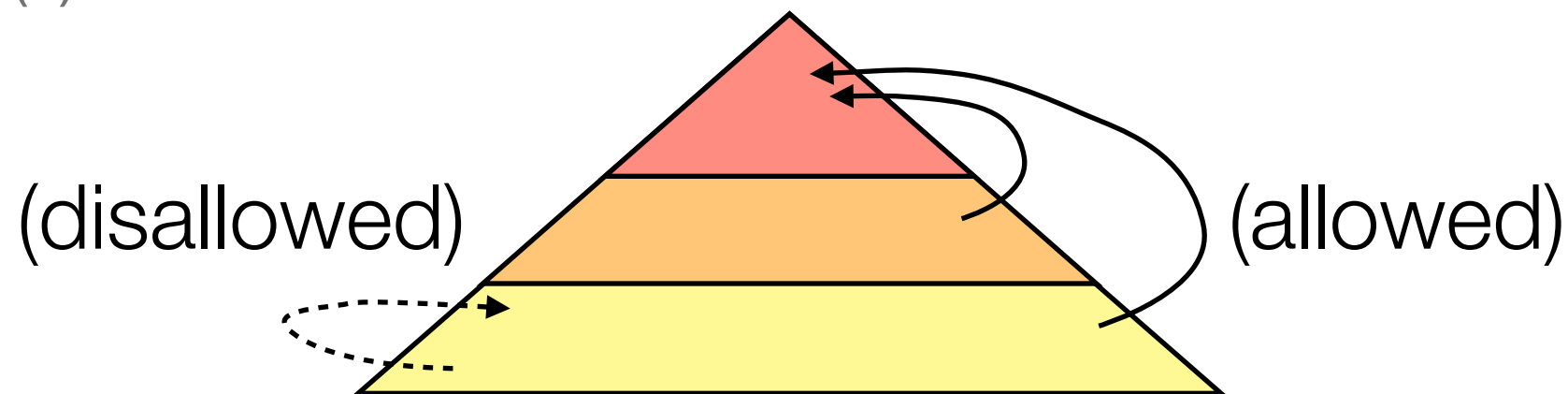


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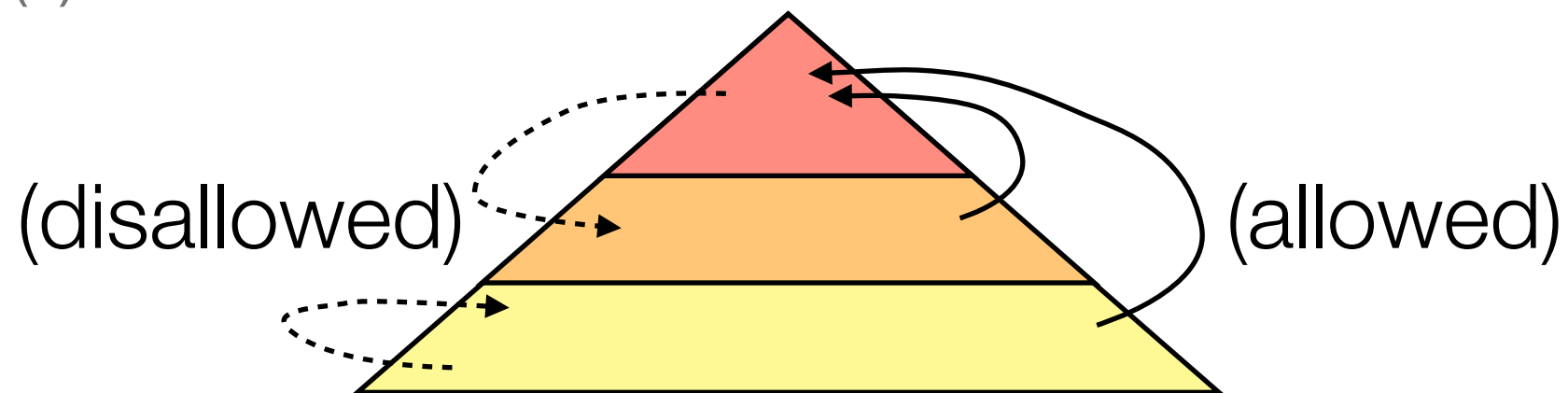
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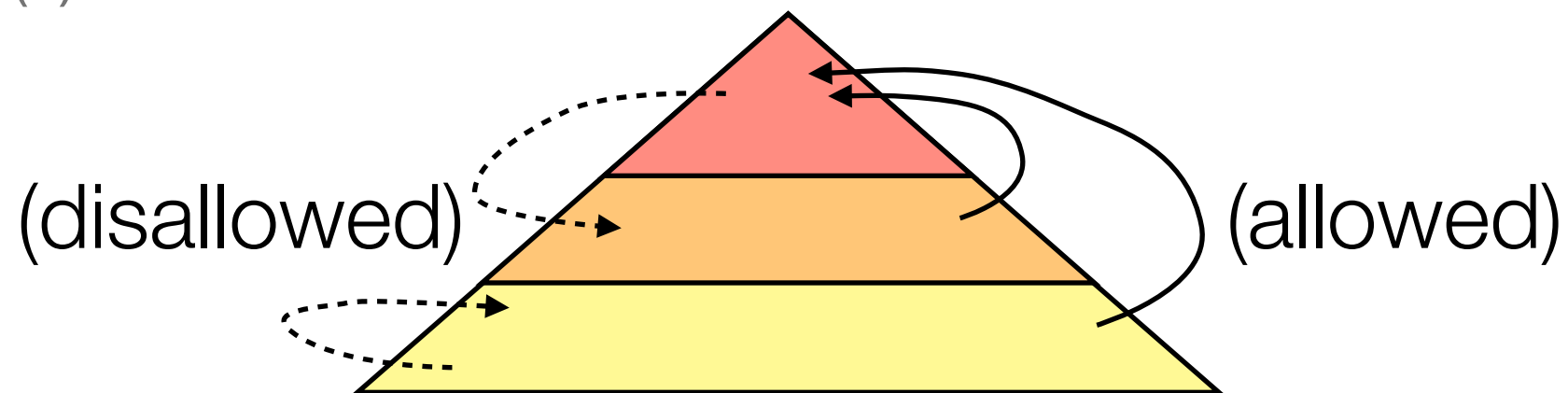
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Also **can't compose more than t transformations**

# Outline

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Definitions

**SNARGs to cm-NIZKs**

Malleable SNARGs  
Boosting to full extractability  
Boosting to CM-SSE

Applying the cm-NIZK

Conclusions

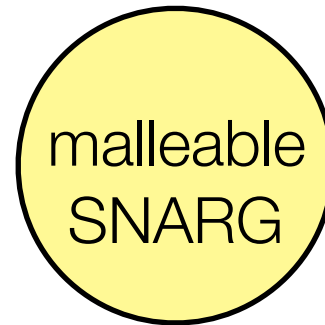
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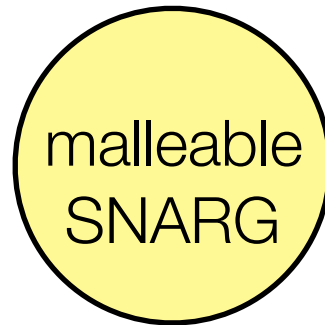
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Our goal: build malleability into SNARGs [BSW12]

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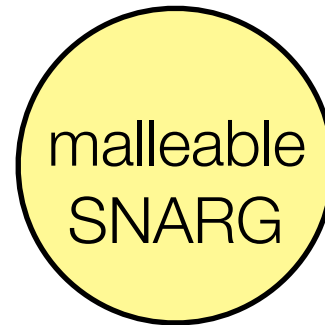


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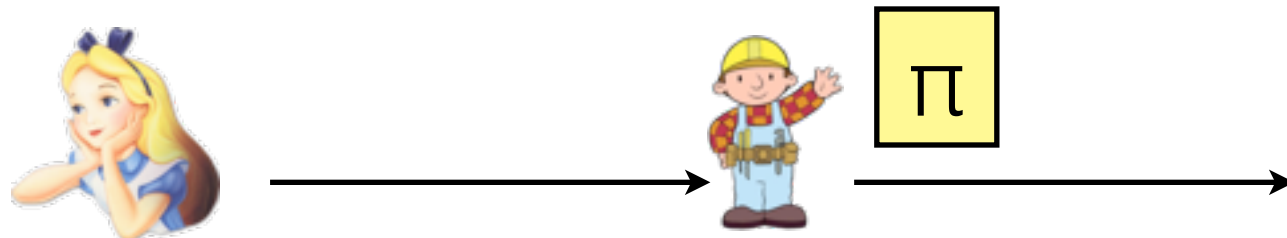
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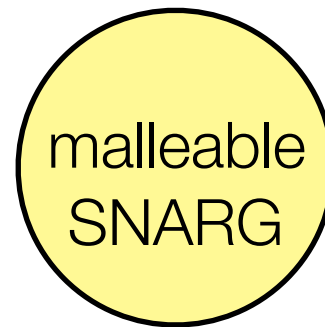
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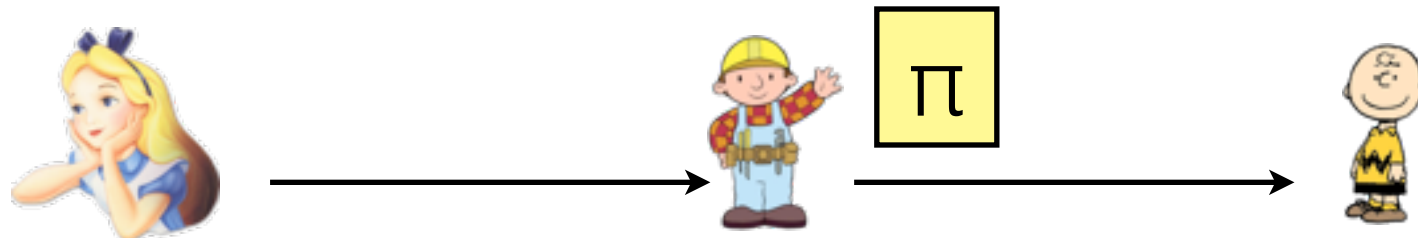
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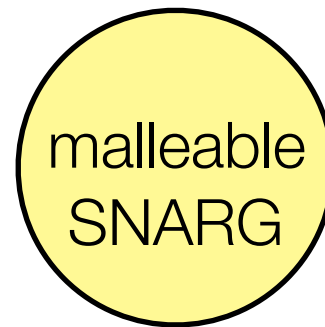
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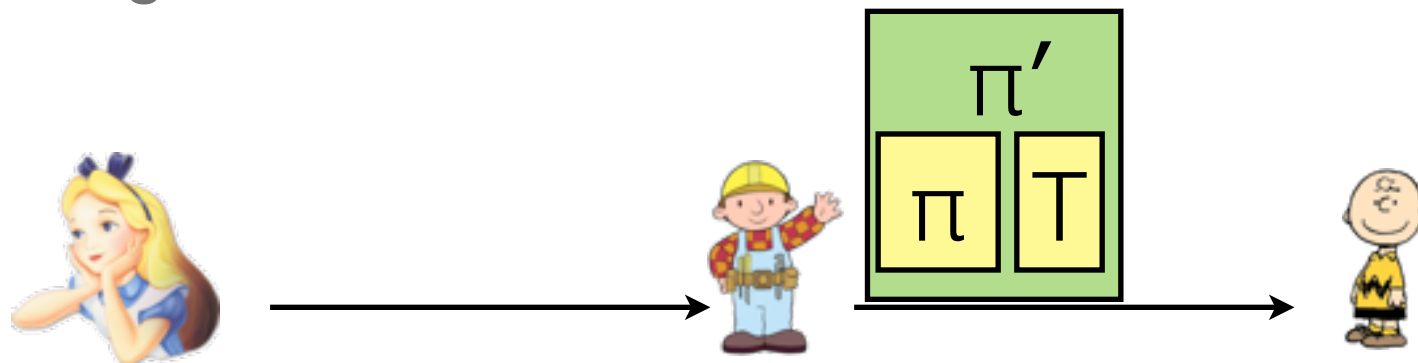
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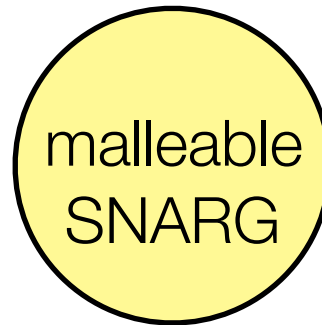
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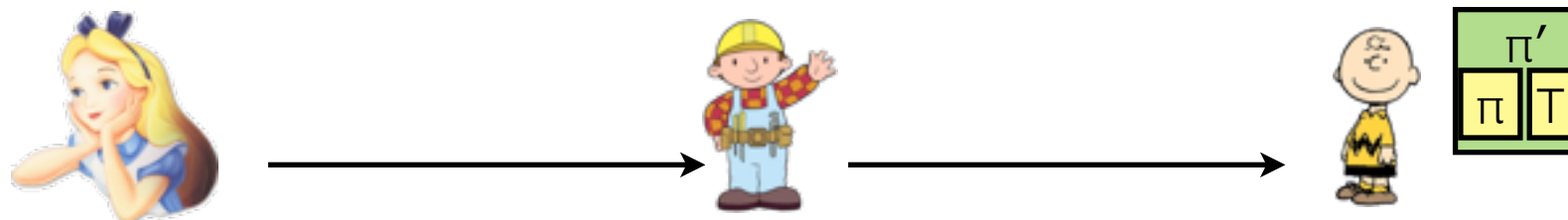
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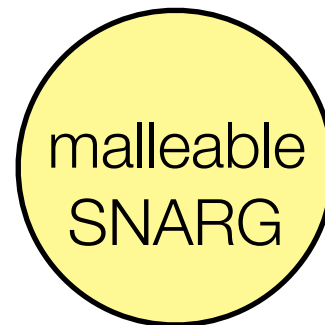
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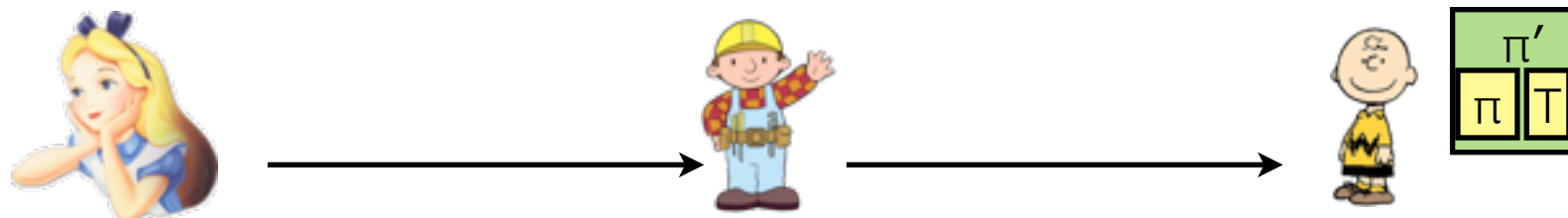
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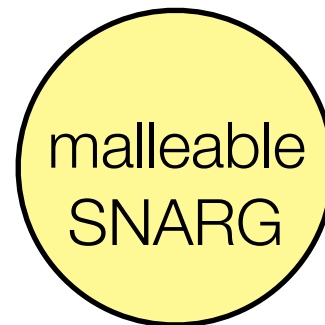
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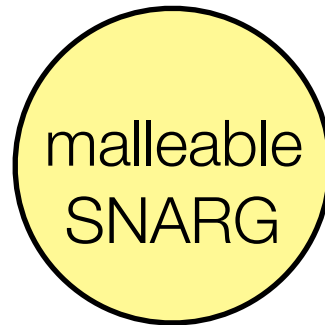


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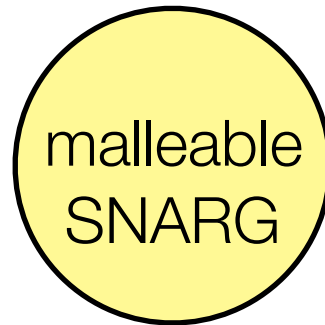
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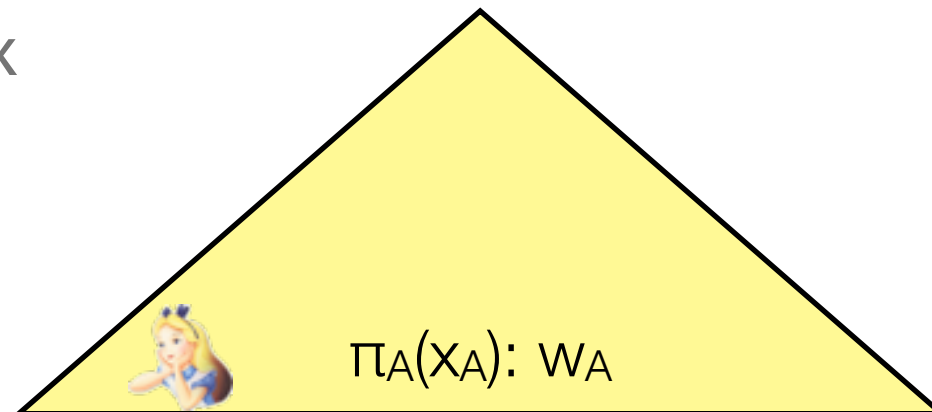
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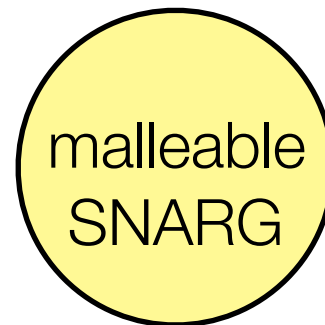


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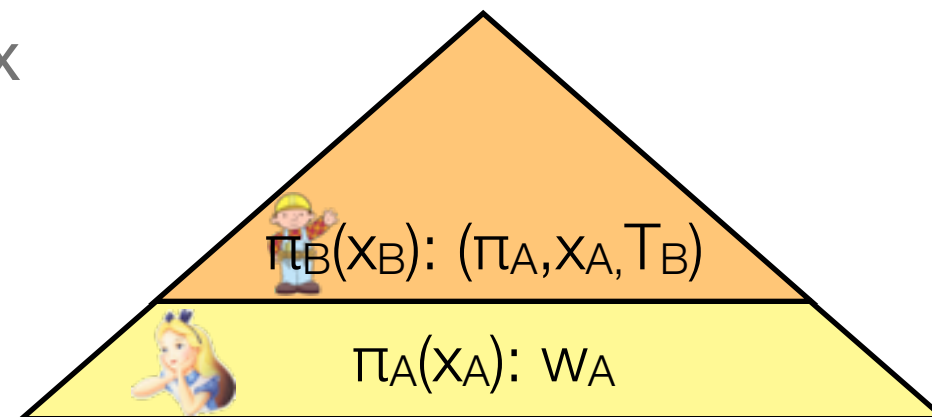


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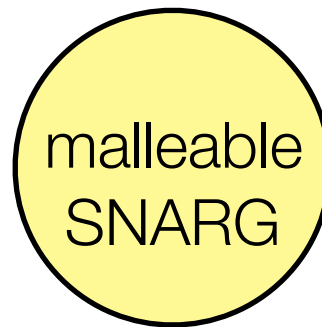


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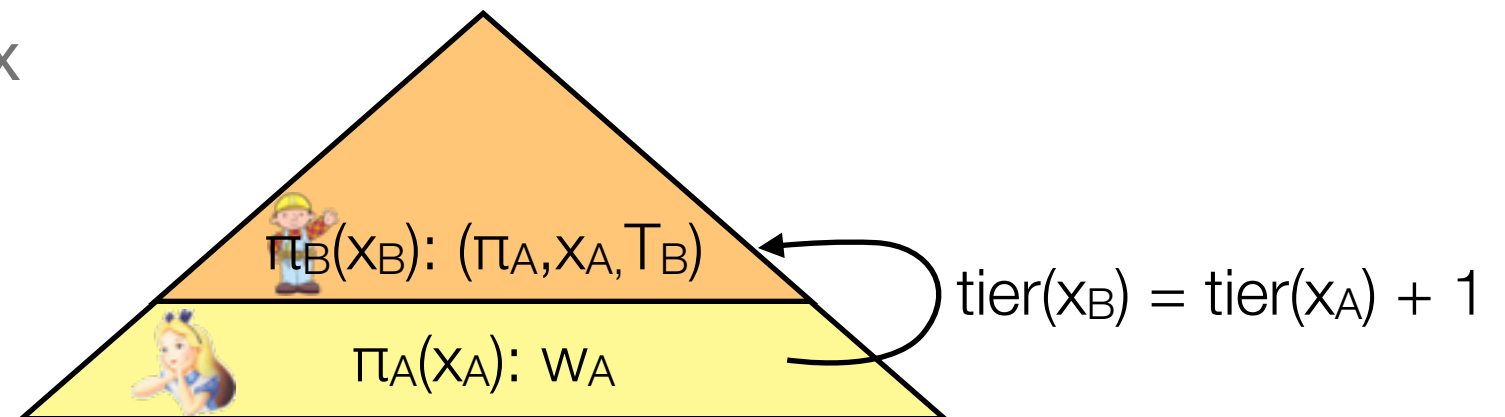


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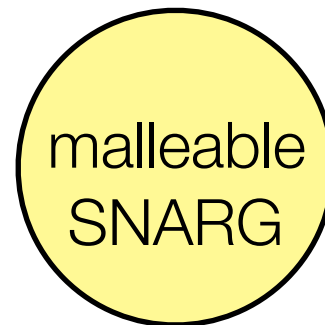


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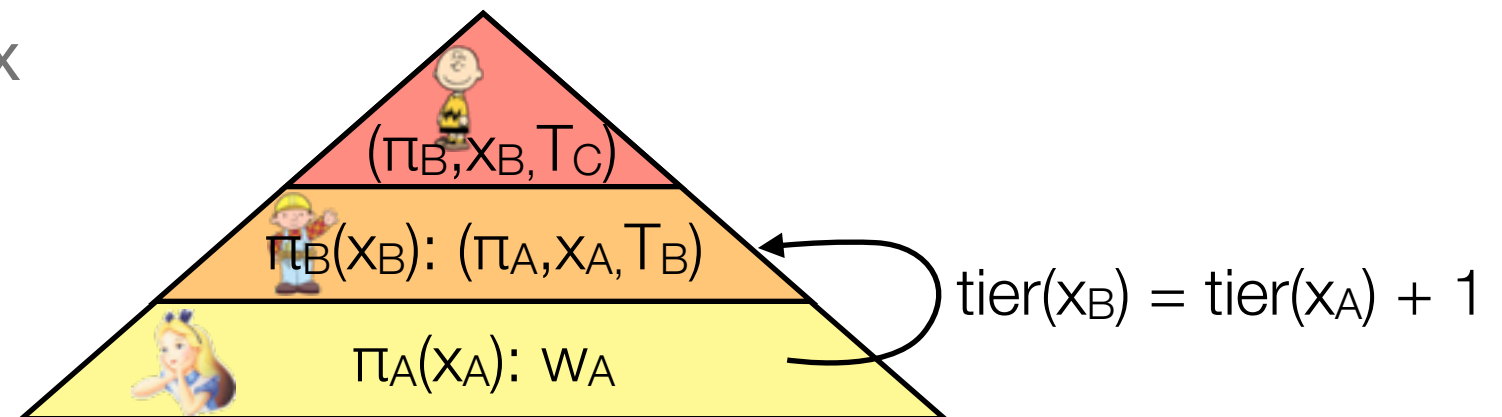


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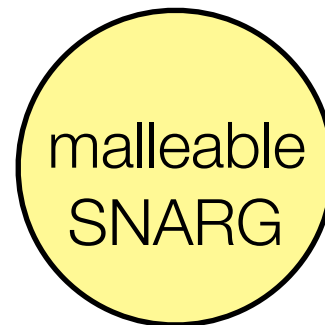


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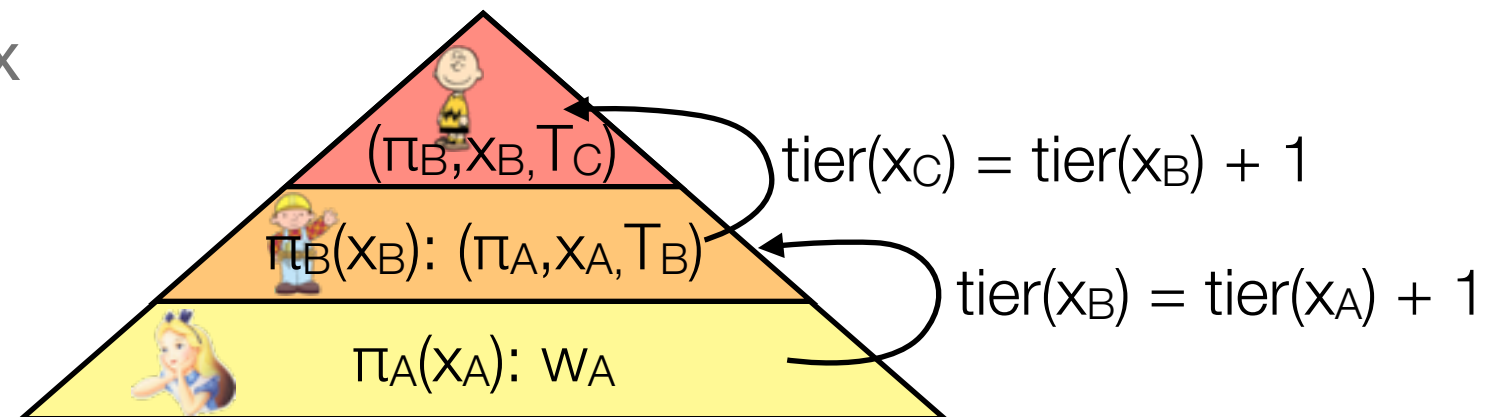


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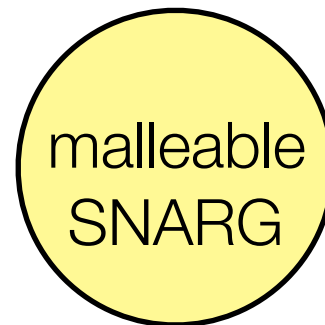


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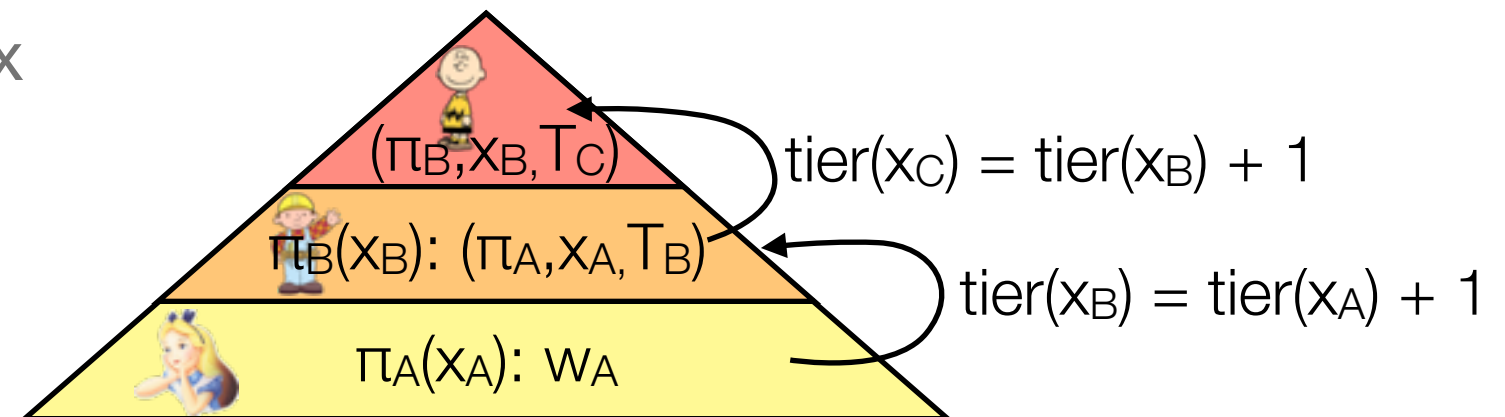


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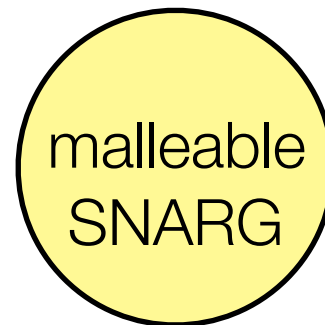
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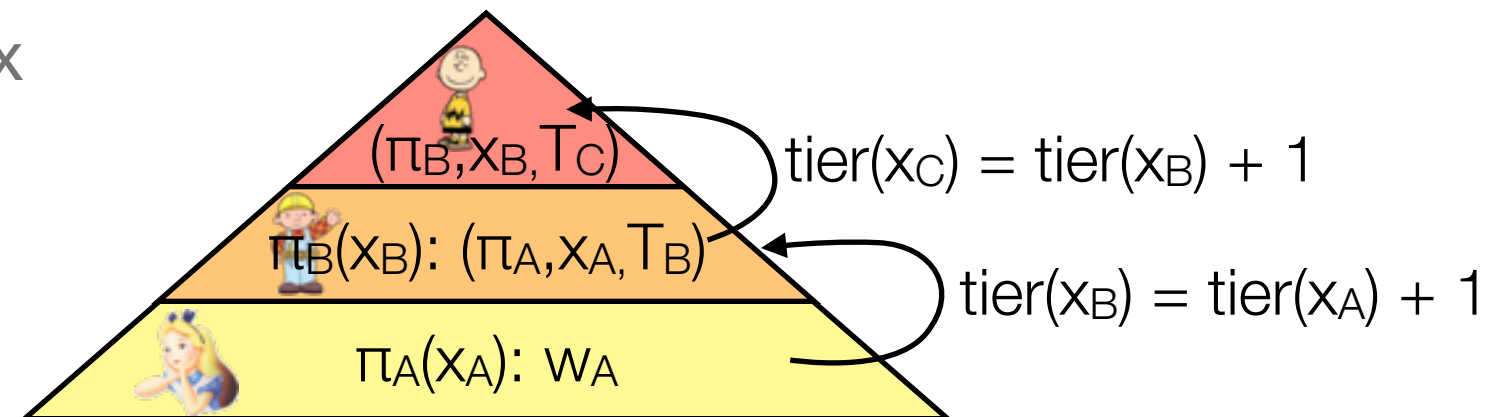
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\*Since extractor might have to “**tunnel down**”  $t$  must be a constant [BSW12, BCCT13] and we use a stronger notion of extraction (consider non-uniform adversaries)

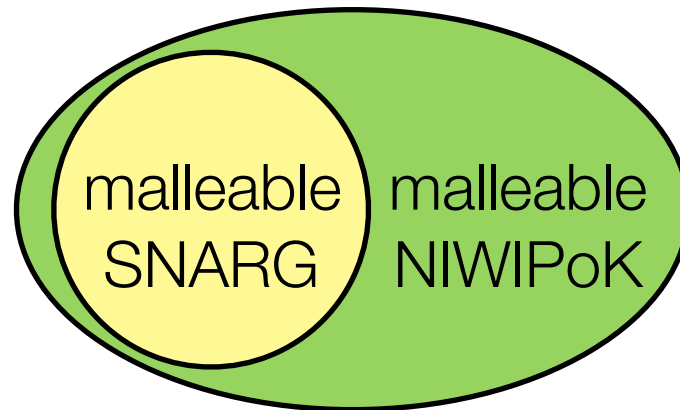
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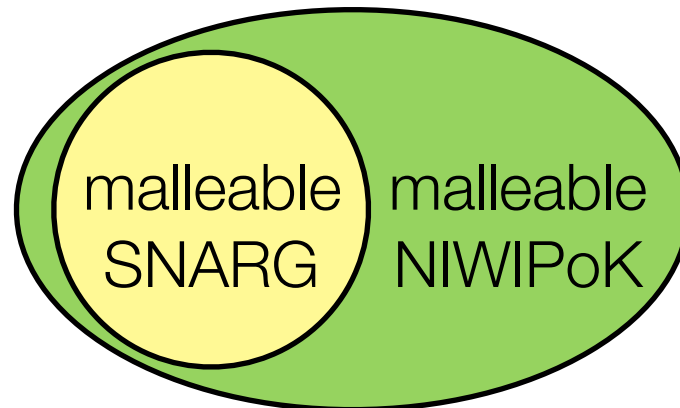
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Our goal: get from adaptive knowledge extraction to stronger soundness

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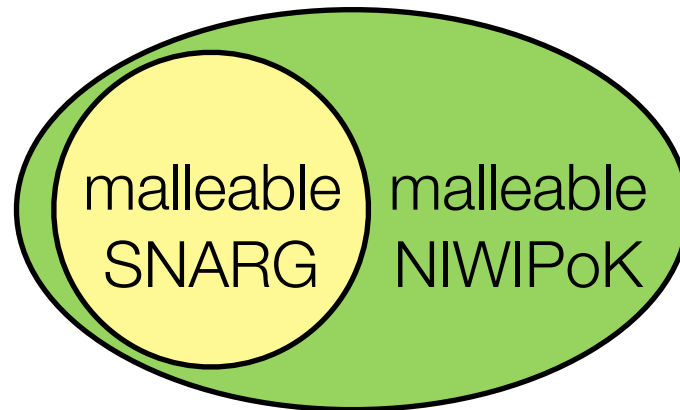


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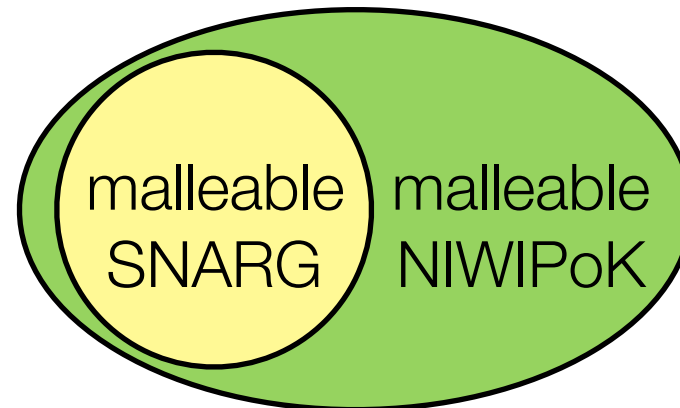
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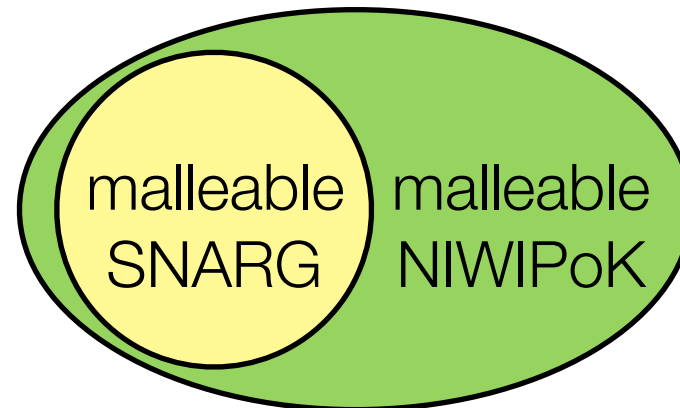
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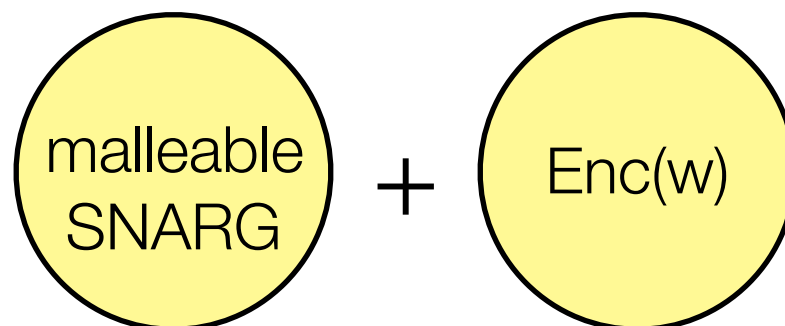
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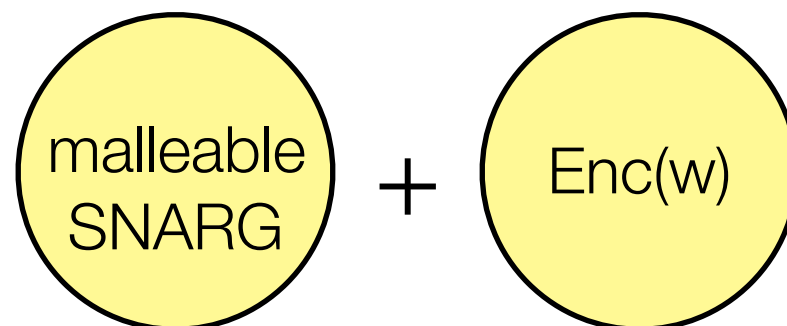
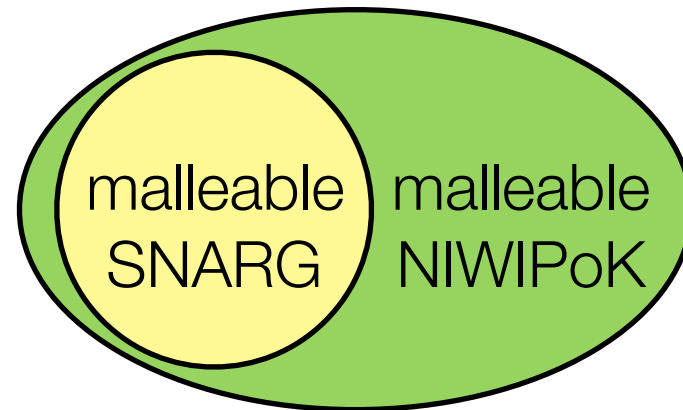
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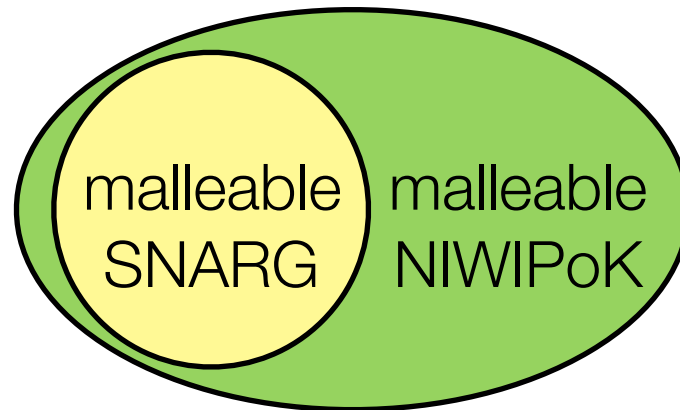
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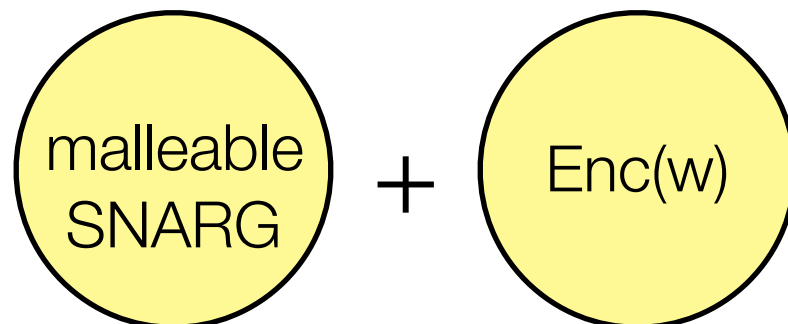


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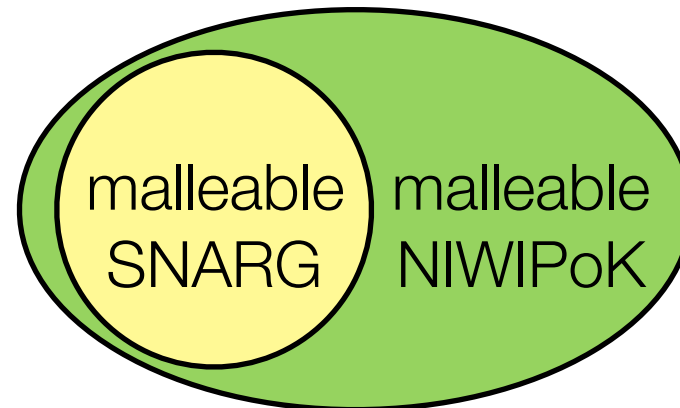


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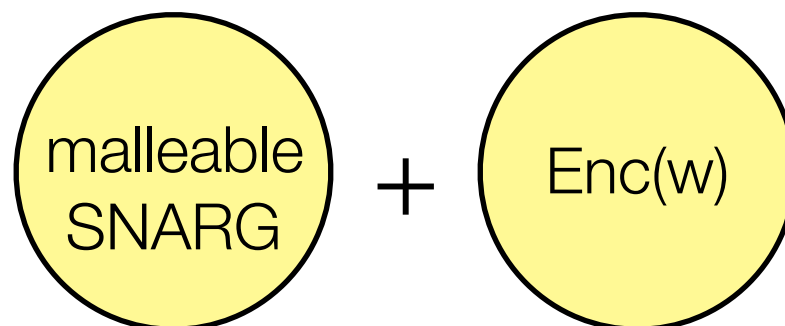
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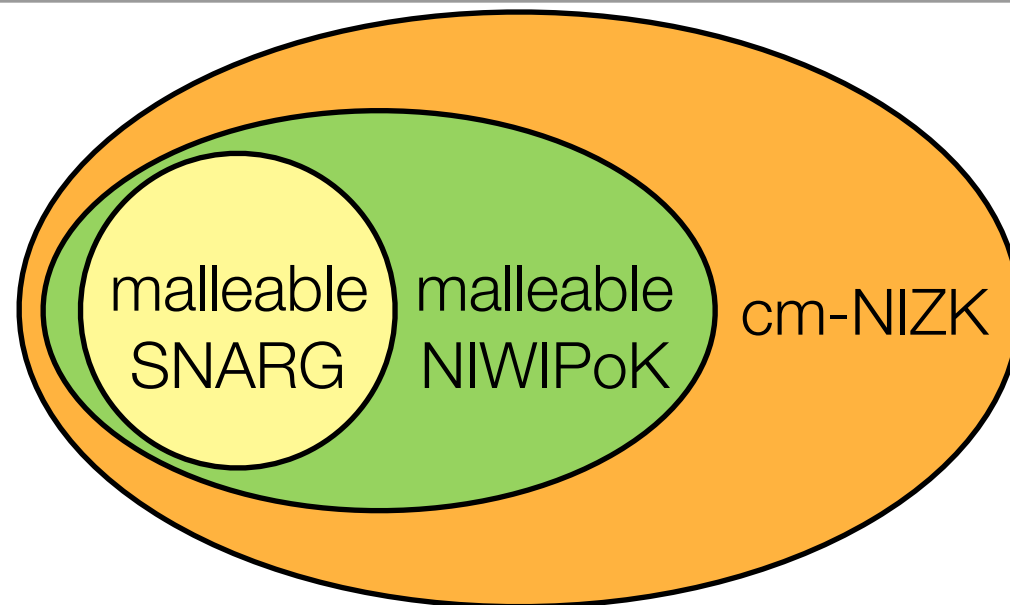
**Extraction is quite simple:**  $\tau_e$  is decryption key, and extractor decrypts, so we never need to use non-black-box SNARG extractor!

If we use a fully-homomorphic encryption scheme, can **preserve malleability** for t-tiered transformations (but we do **lose succinctness**)



# Boosting to CM-SSE

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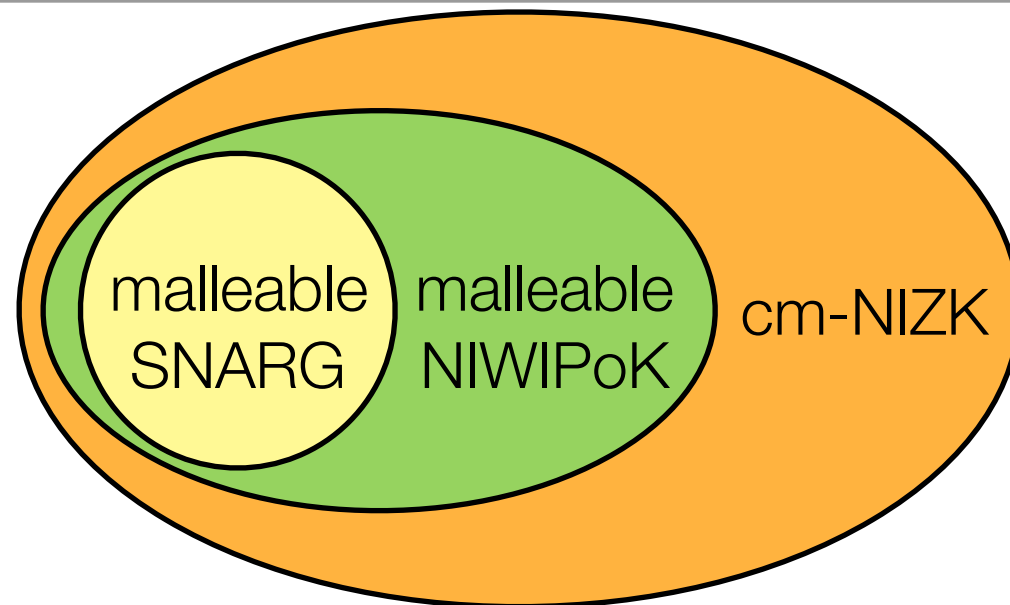


Our goal: preserve malleability with respect to  $t$ -tiered transformations

Essentially amplify [CKLM12] construction; don't assume certain transformations (e.g., the identity) are allowable

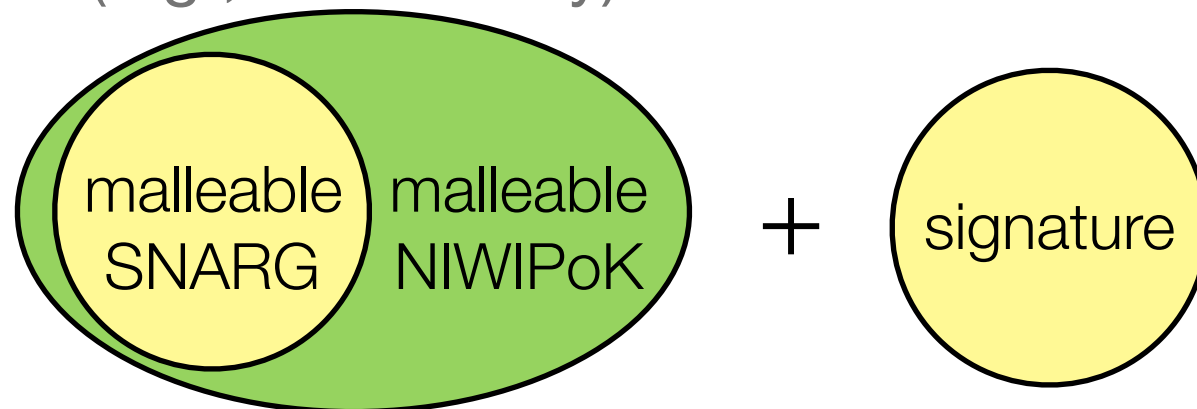
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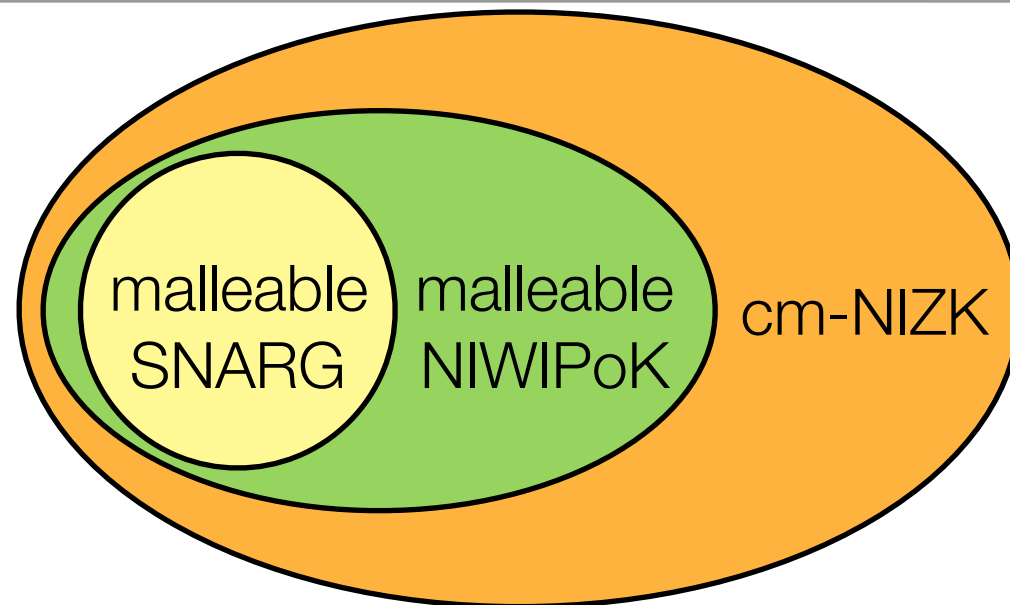
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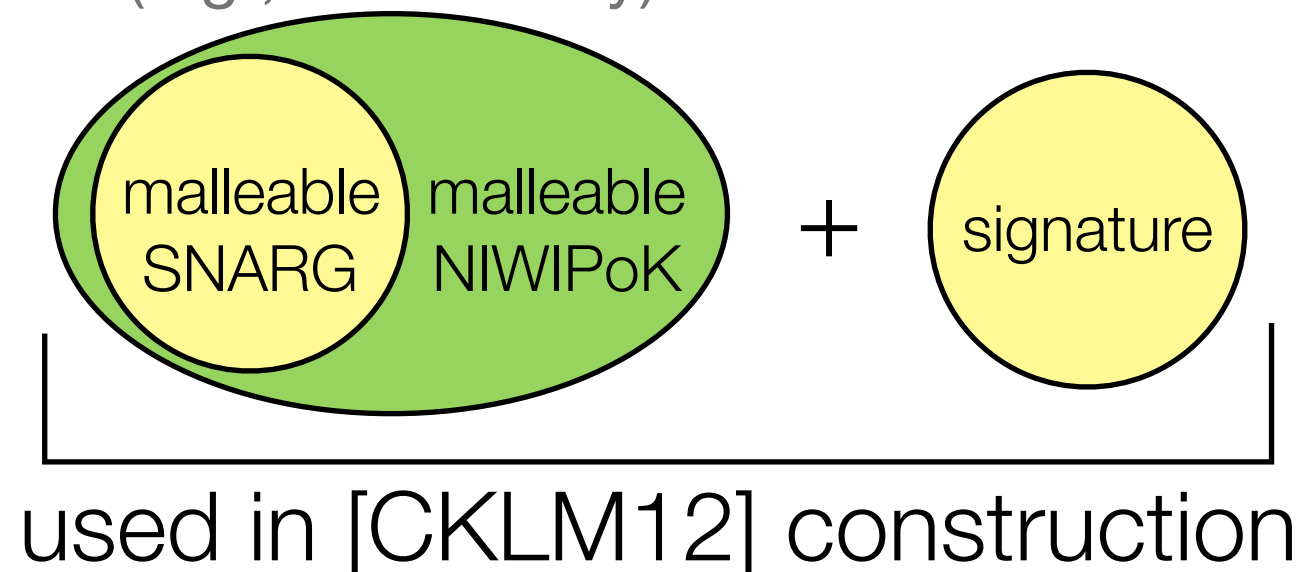
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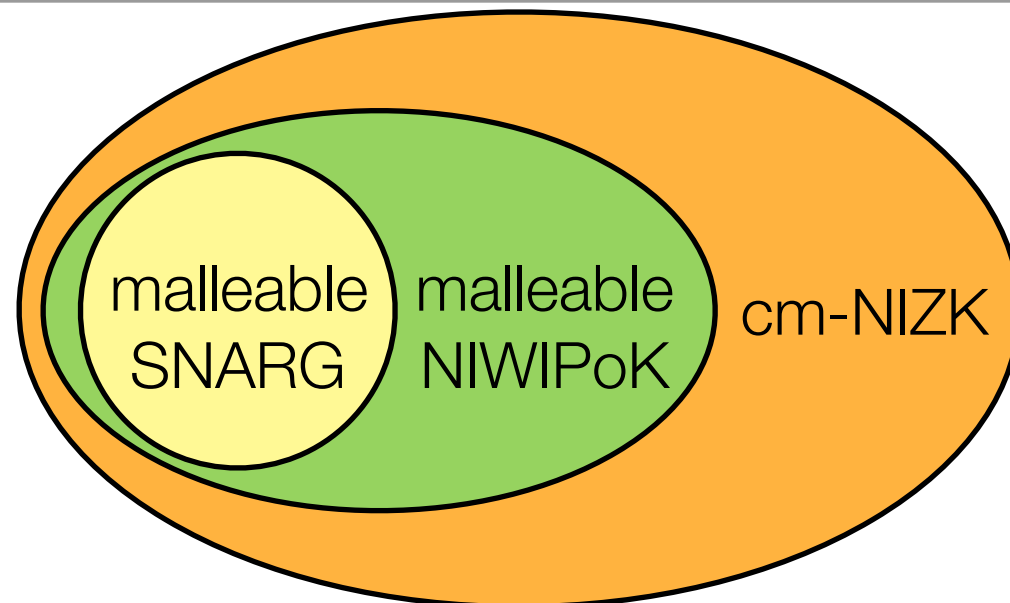
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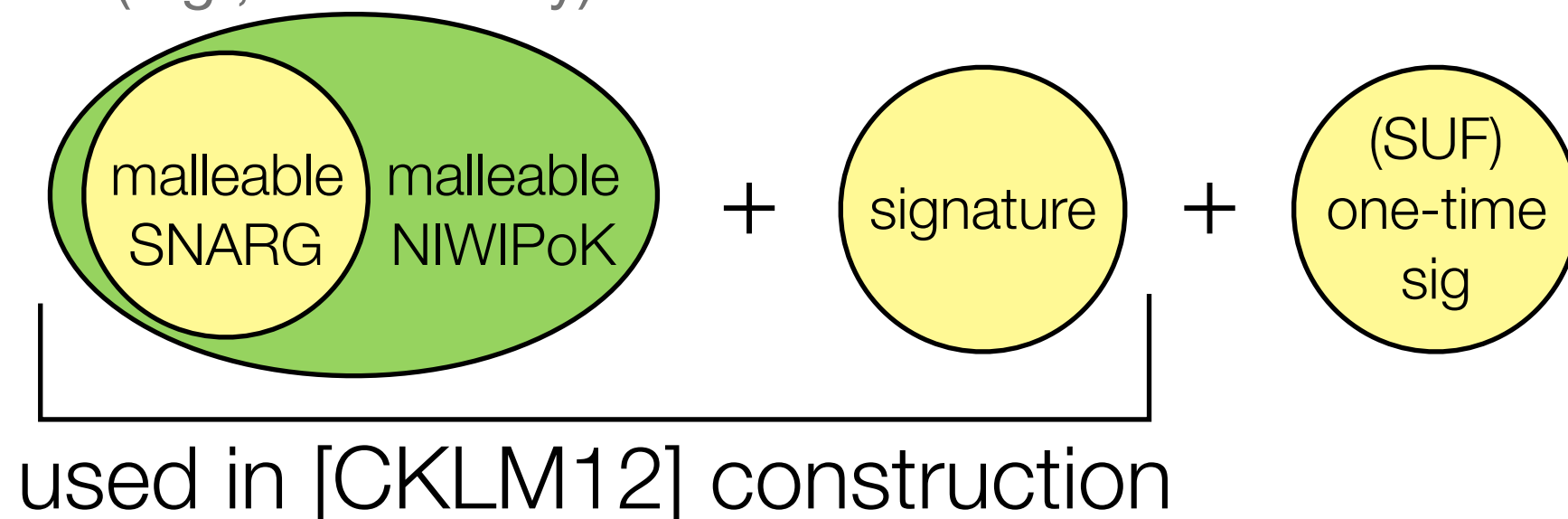
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# Outline

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Definitions

SNARGs to cm-NIZKs

Applying the cm-NIZK

Conclusions

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To instantiate a cm-NIZK, had to therefore **jump through a lot of hoops!**

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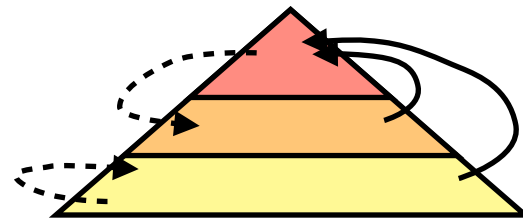


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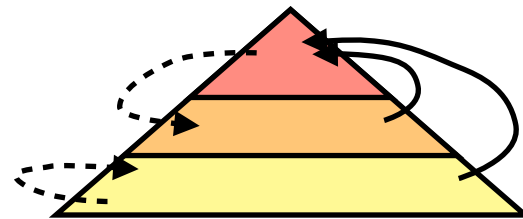
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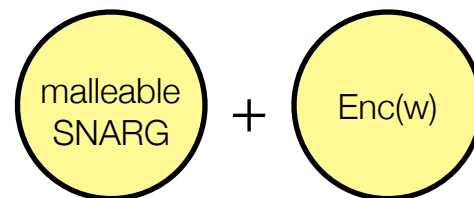
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- **Step 2**: Give **instantiation** for encryption scheme depending on how much malleability you want (for shuffle: multiplicatively homomorphic encryption)

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**Thanks!**  
**Any questions?**