ZKPDL: A Language-Based System for Zero-Knowledge Proofs and Electronic Cash

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Crypto



 $R \leftarrow H(pk_{\mathcal{M}} || contract) ;$ $x_1, x_2, x_3 \leftarrow \mathbb{Z}_q ;$ $y \leftarrow \phi(x_1, x_2, x_3) ;$ $S' \leftarrow F_s(J)g^{x_1} ;$ $T' \leftarrow g^u F_t(J)^R g^{x_2} ;$



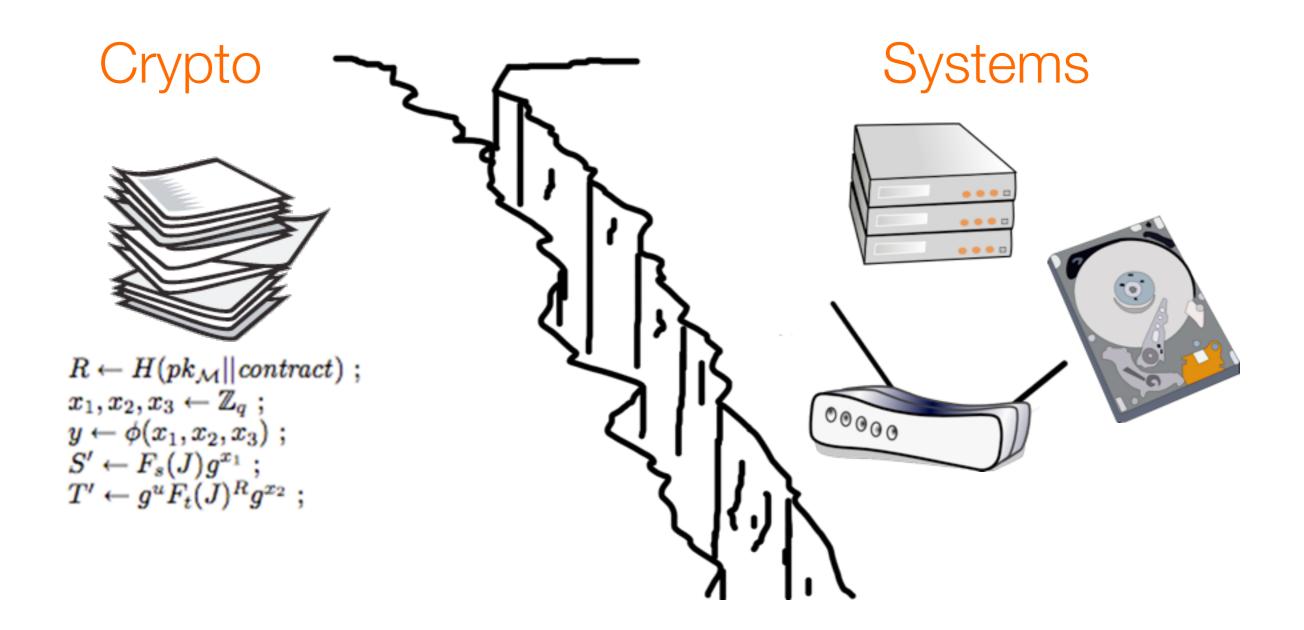
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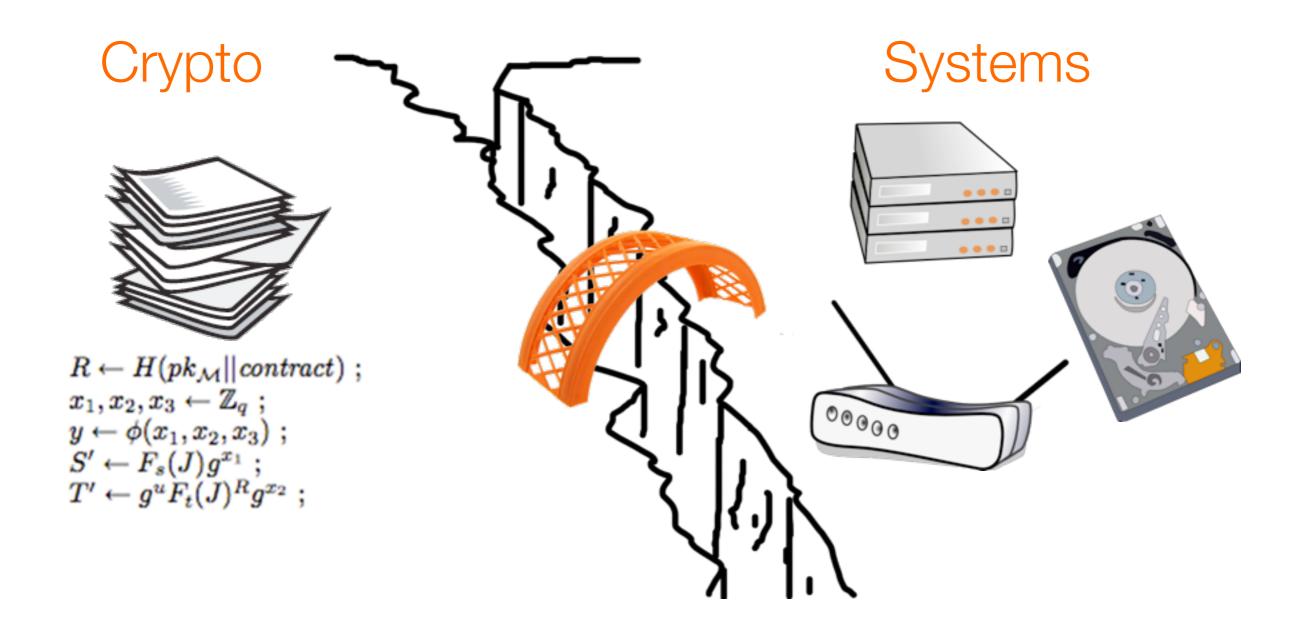


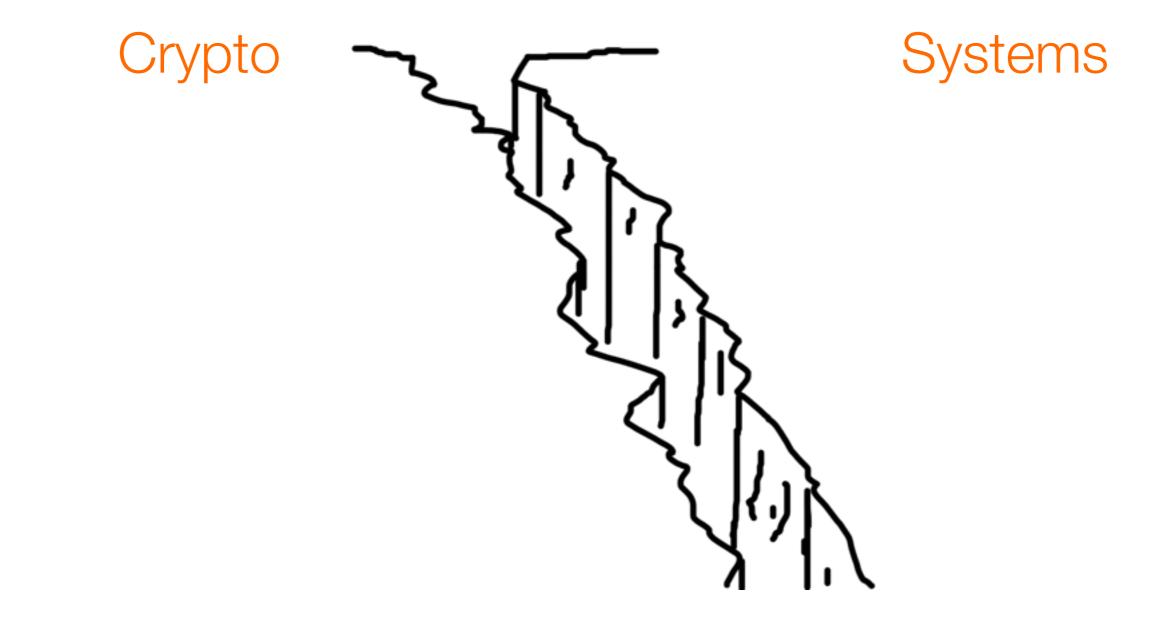
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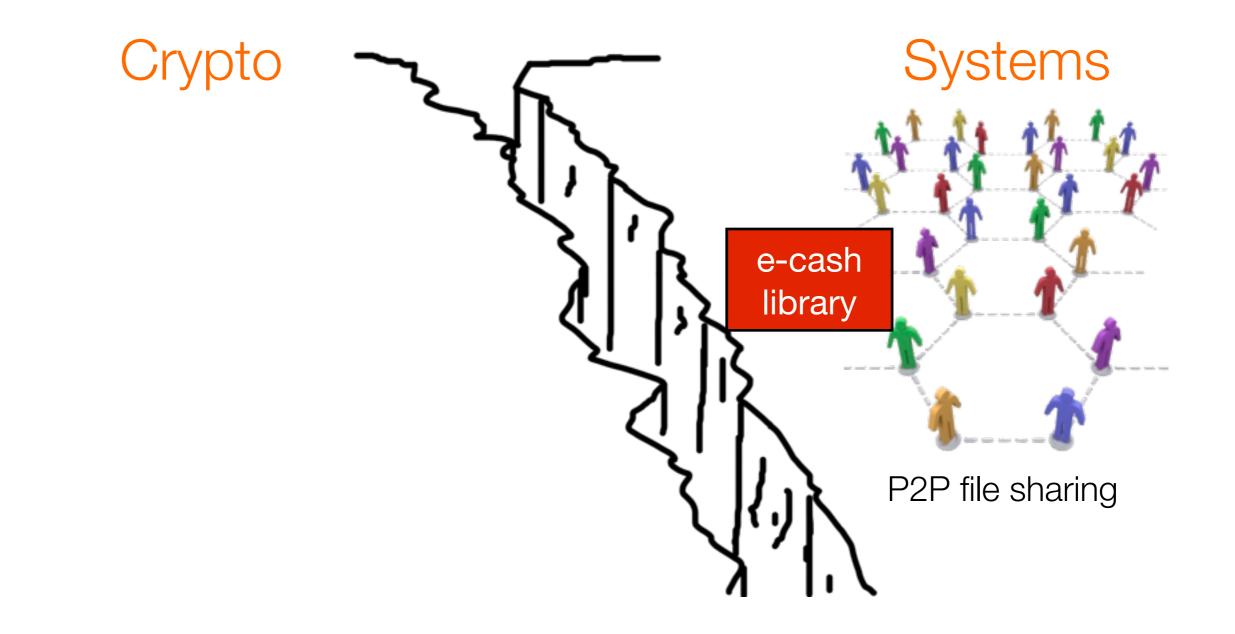


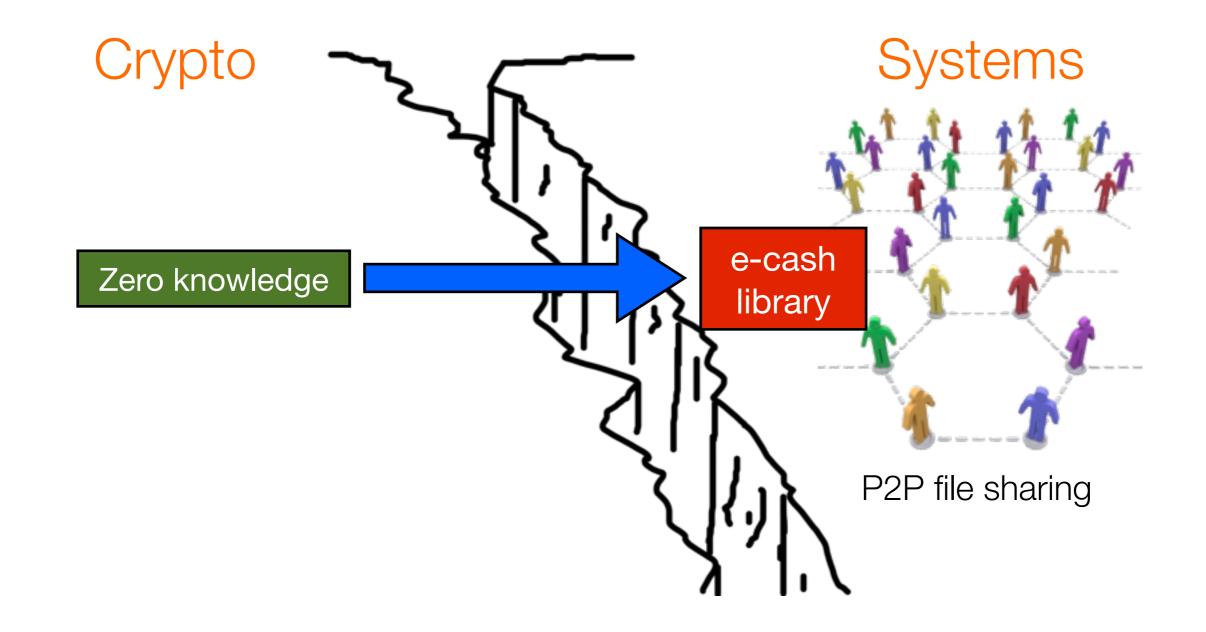


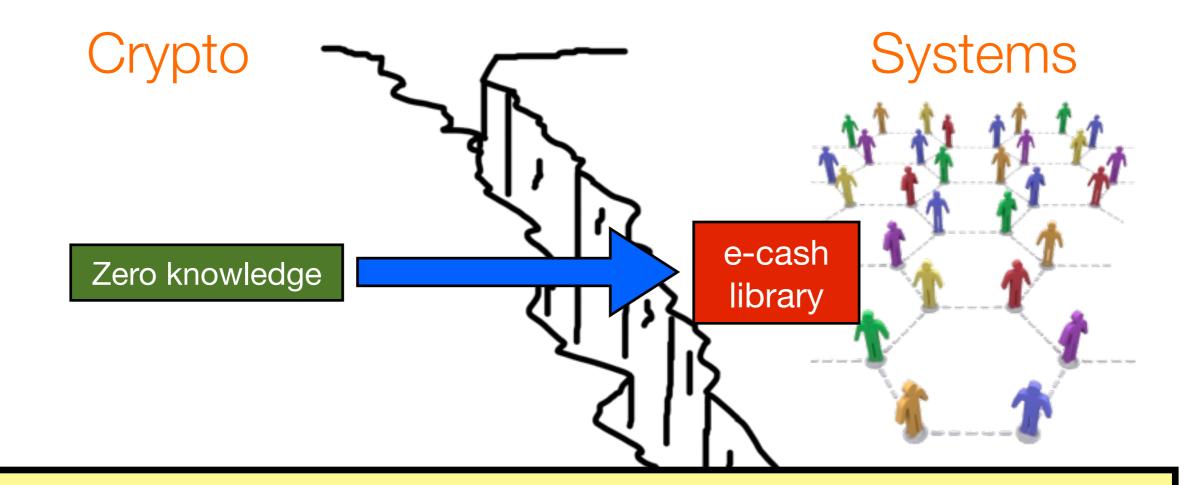












Wrote language for zero-knowledge proofs

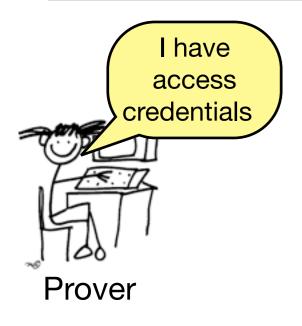
Removes obstacle, easy to translate from description to implementation

Wrote library for e-cash using this language/interpreter framework



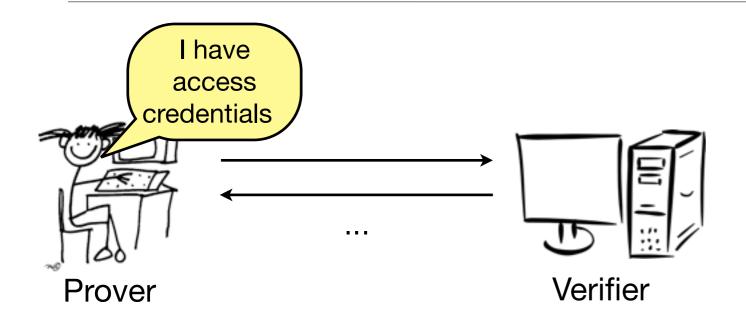


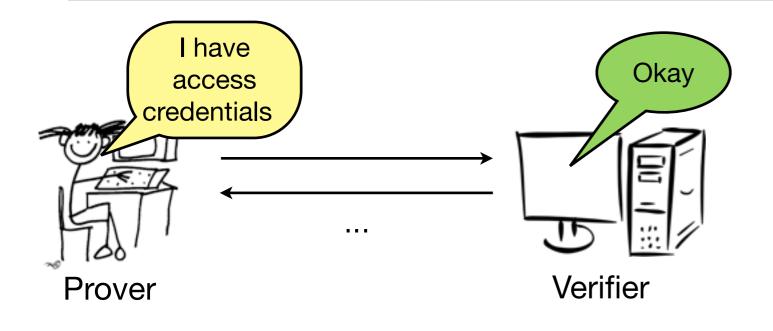
Verifier

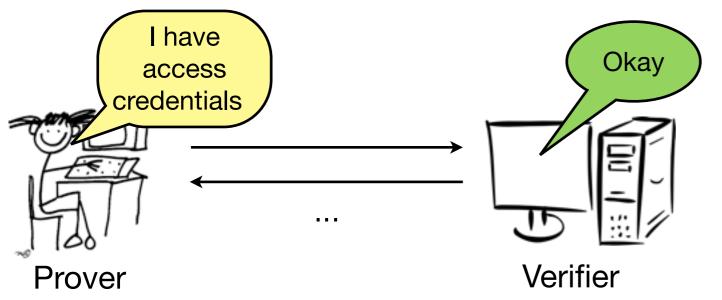




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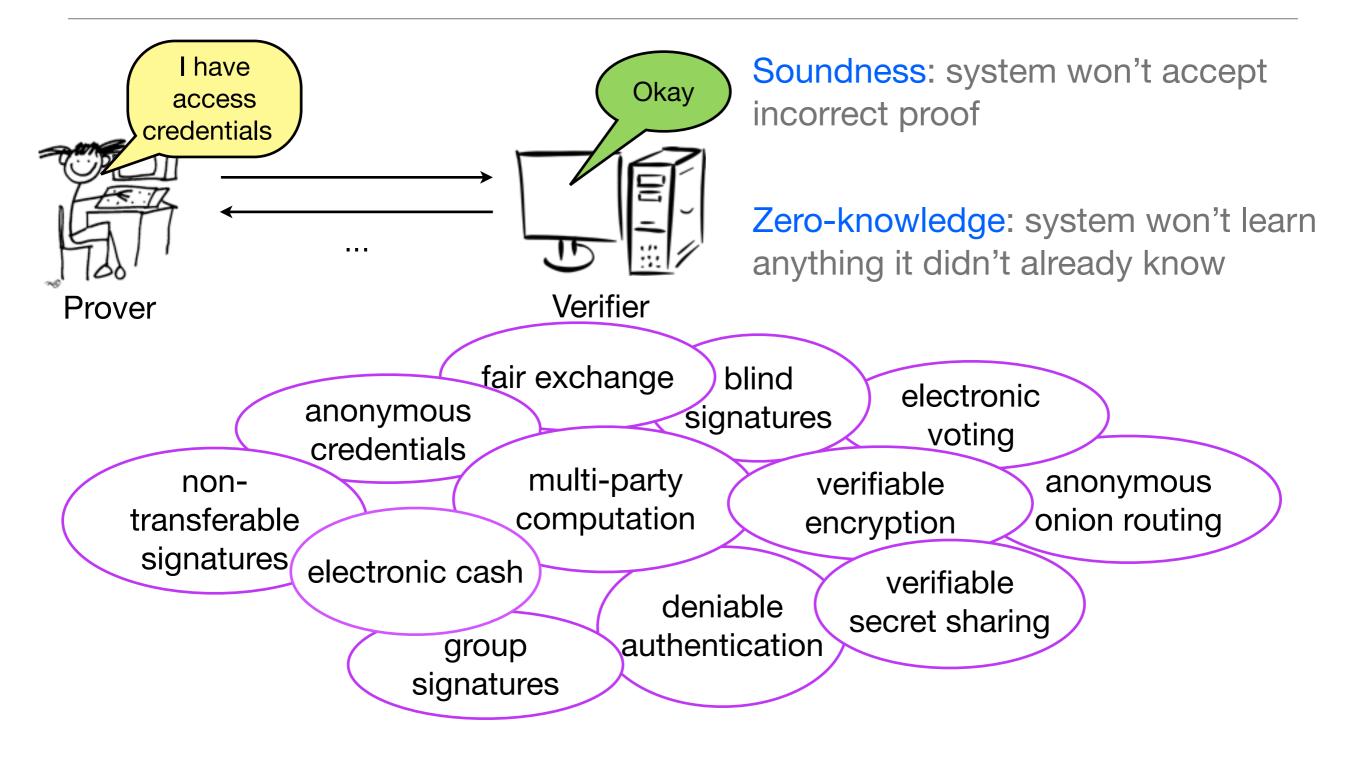


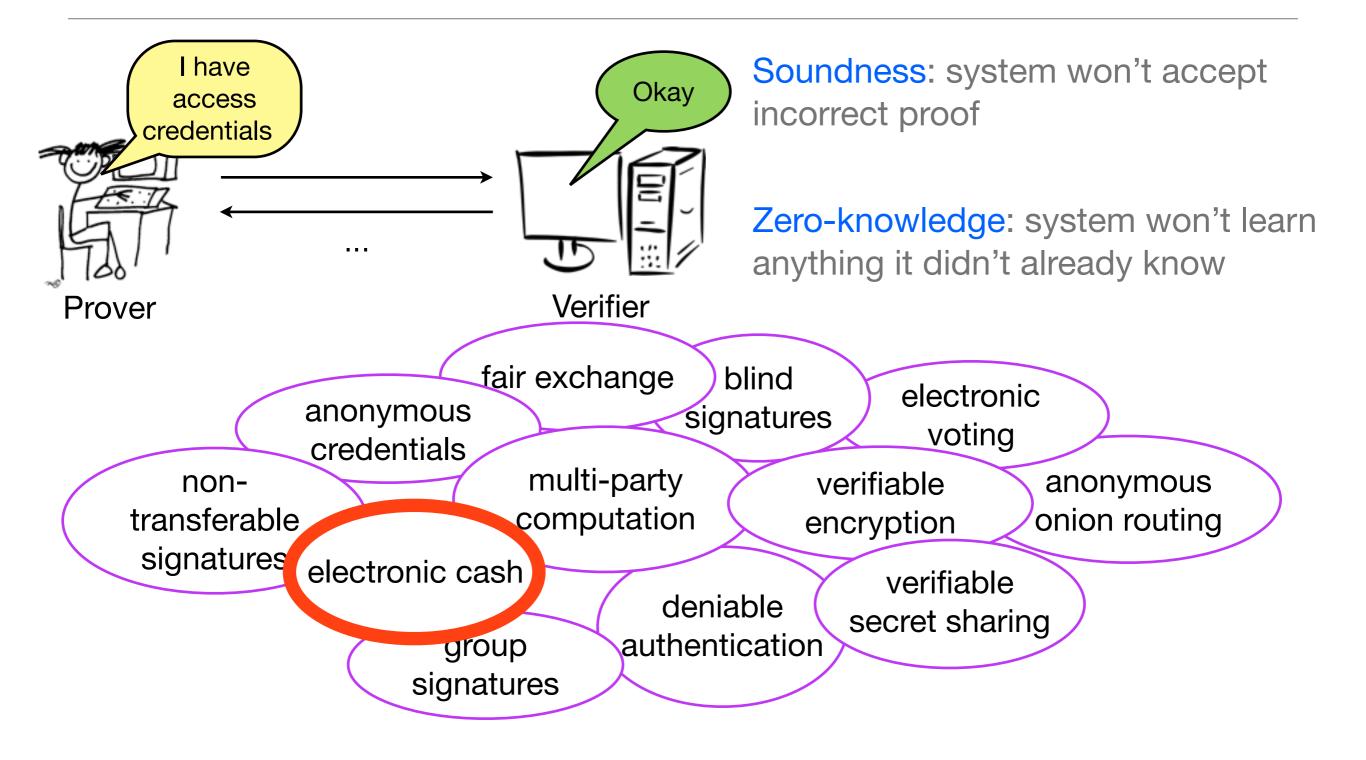


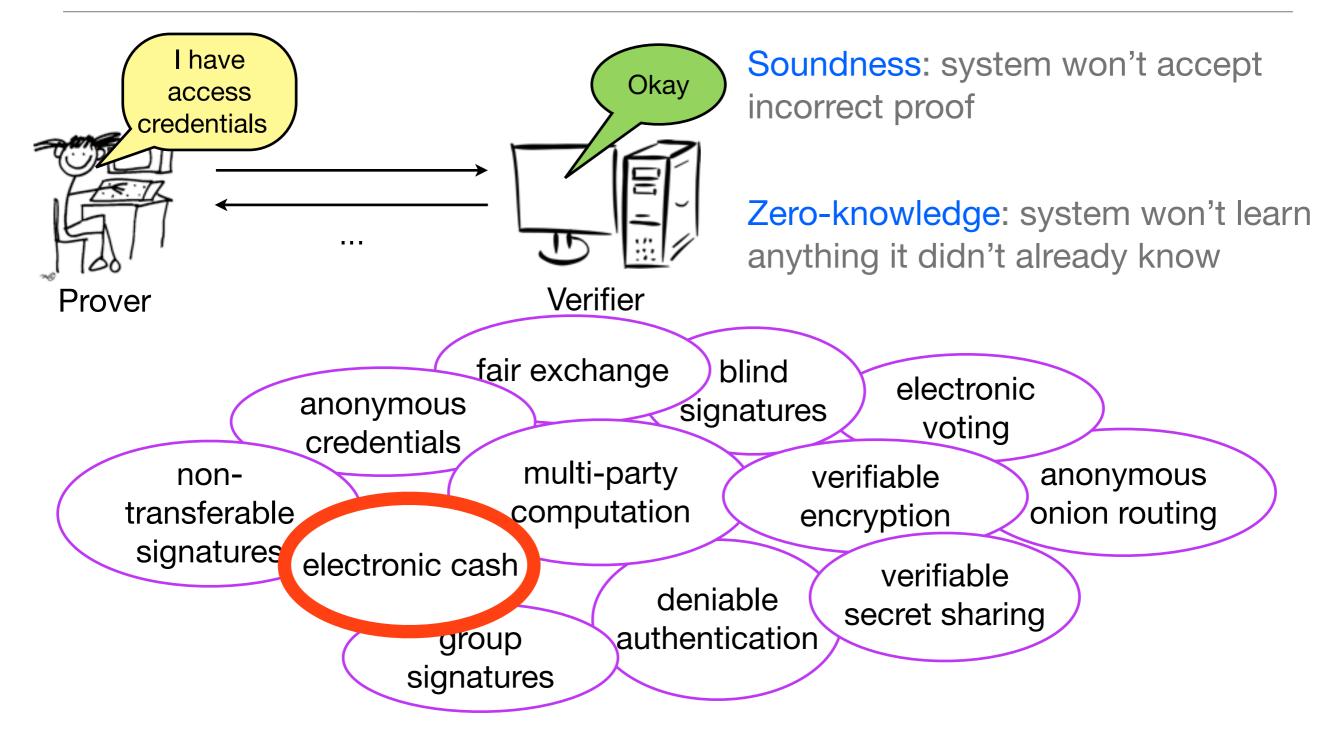


Soundness: system won't accept incorrect proof

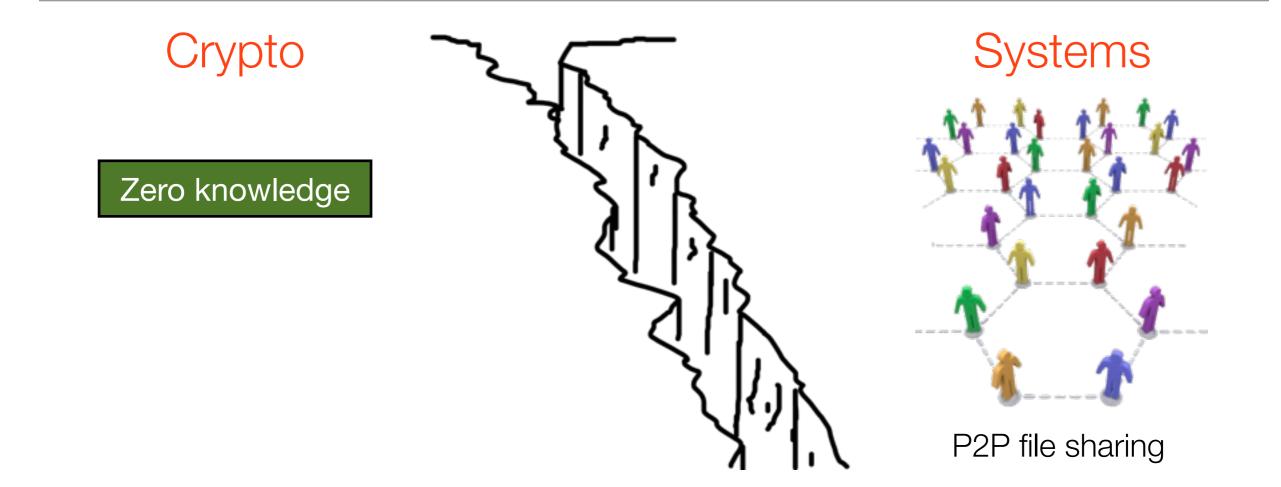
Zero-knowledge: system won't learn anything it didn't already know

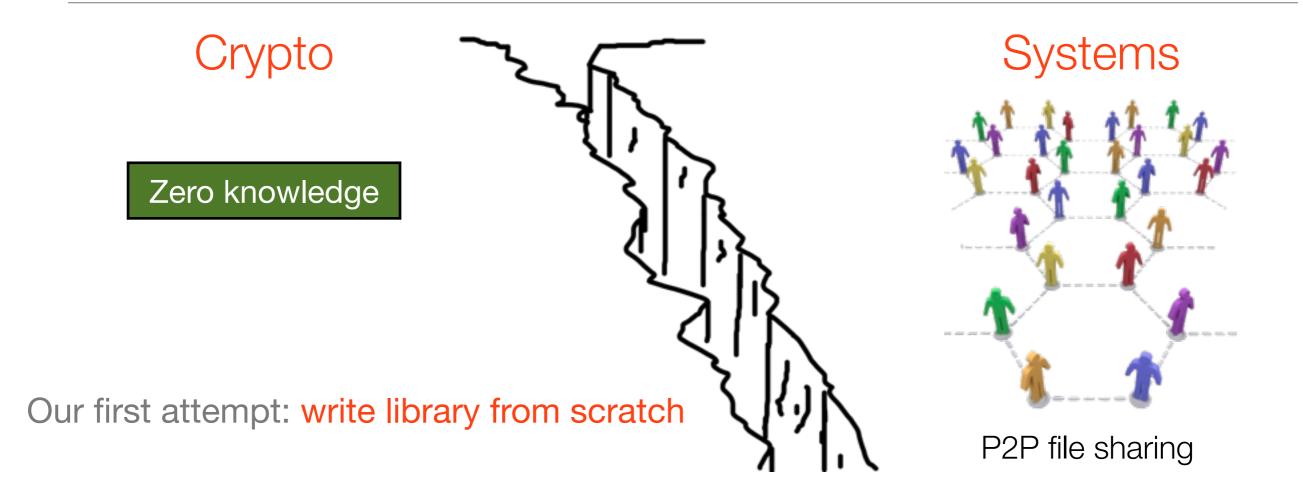


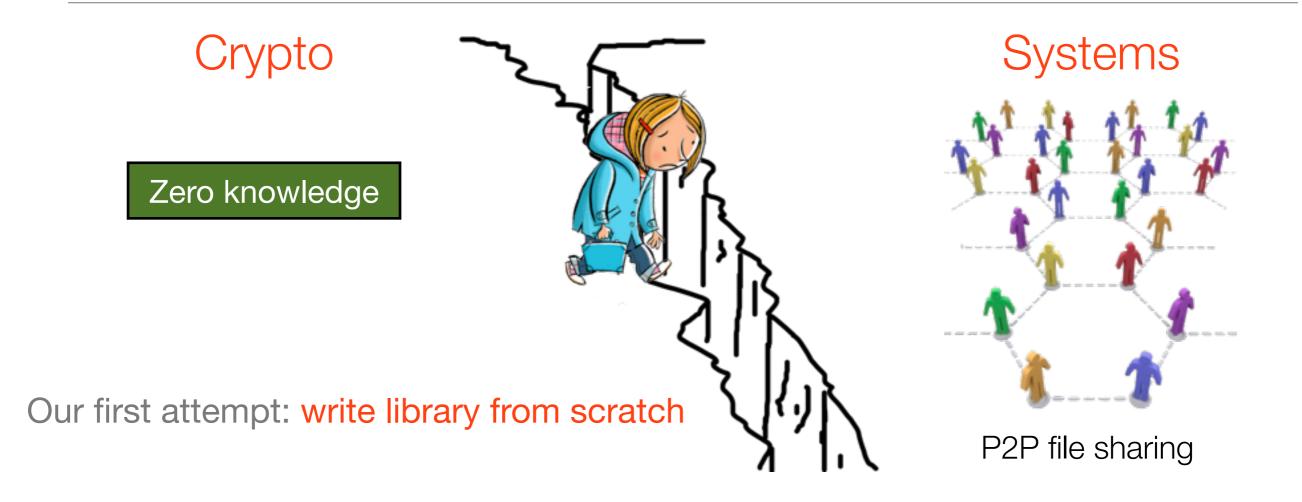


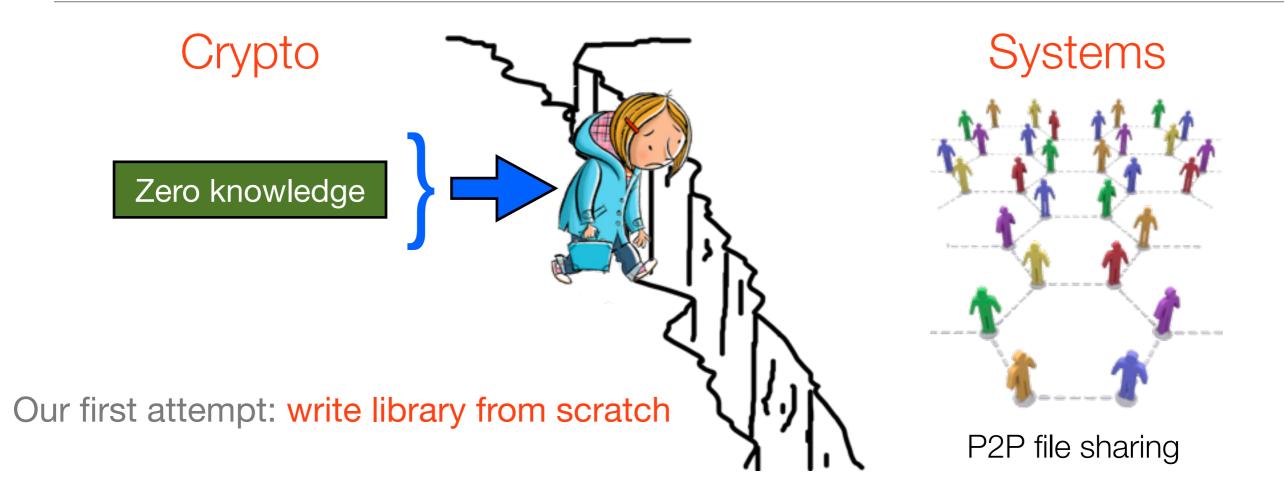


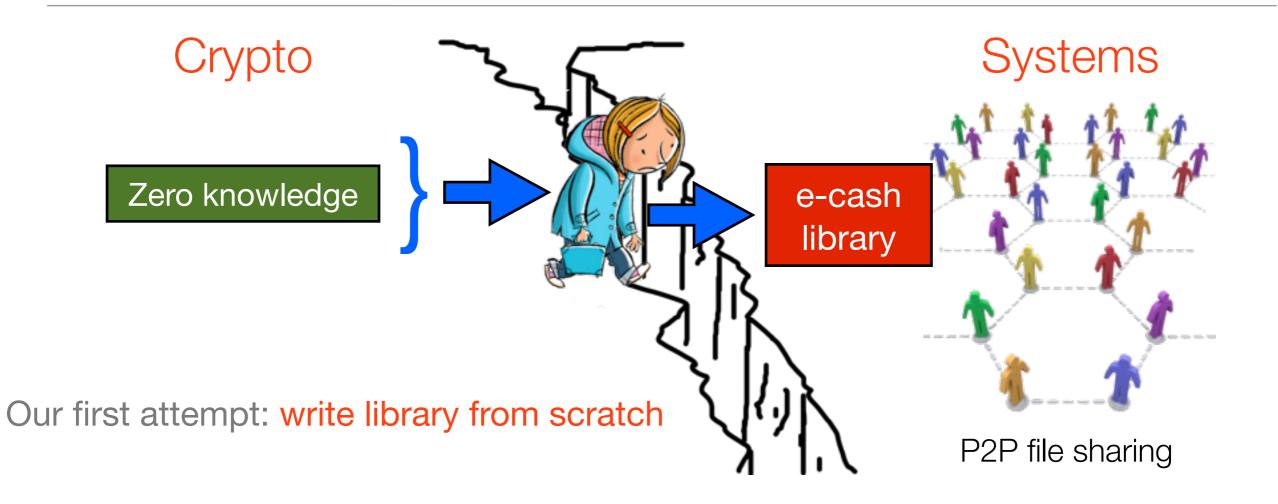
Zero-knowledge proofs have applications, but can be complex

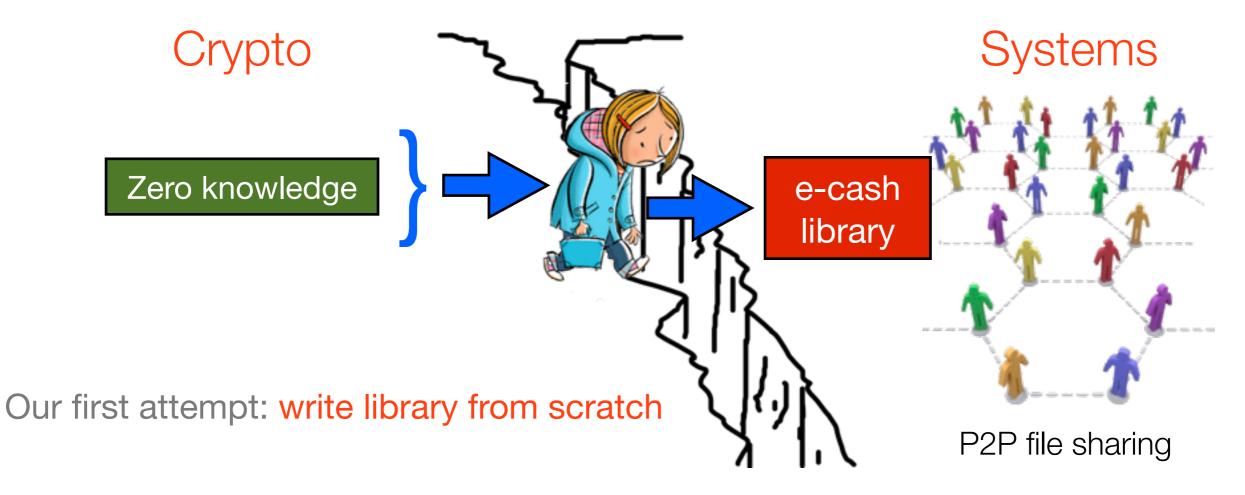












- Not reusable
- Time-consuming
- Error prone

• Lesson learned: even though you know the math, coding can get messy

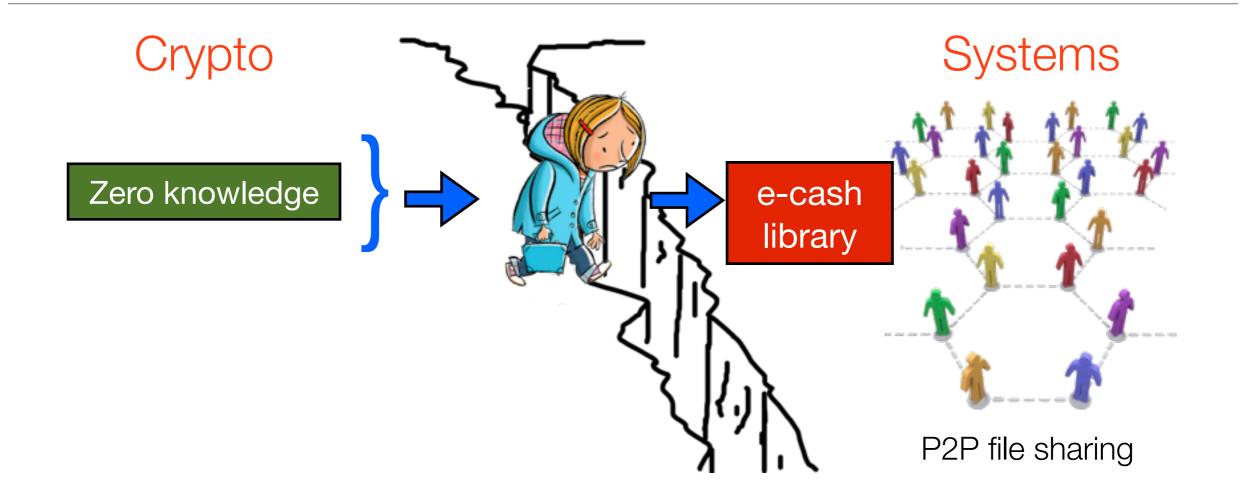
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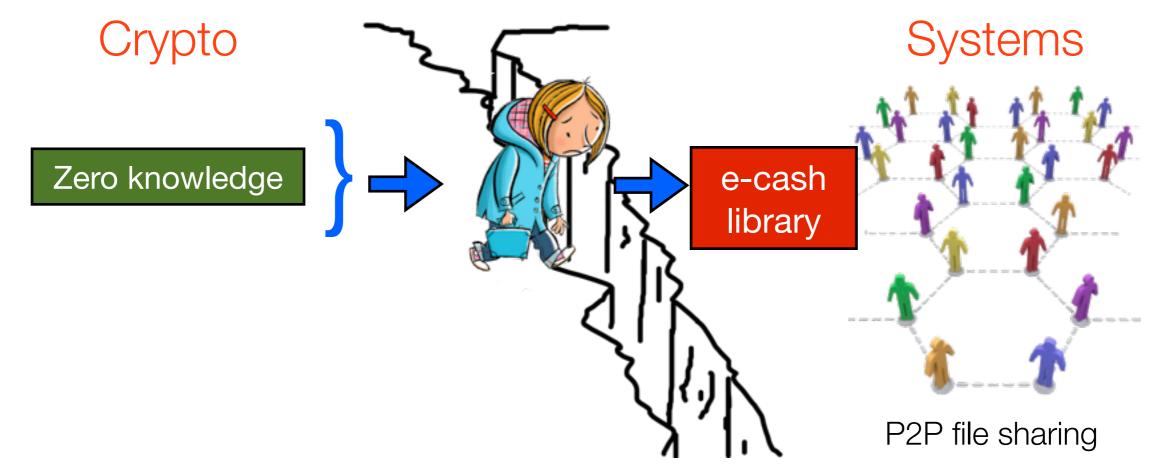
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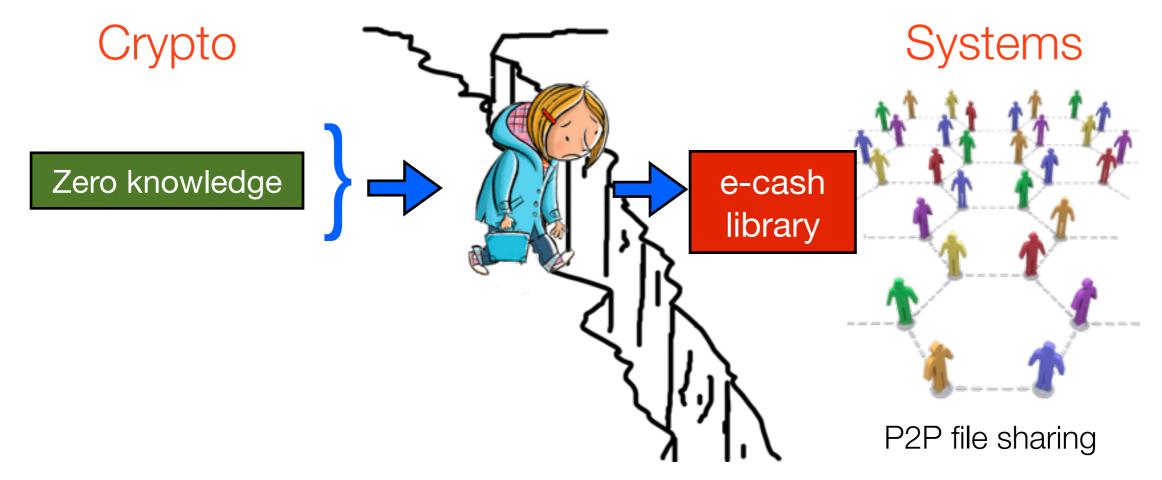
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• Functionality is there, but not easy to use



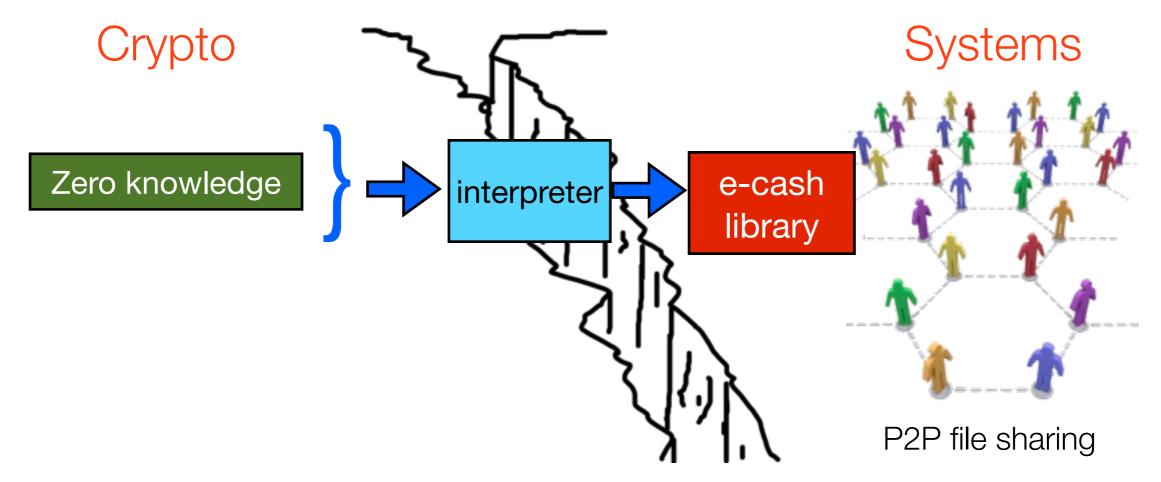


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High-level language, goal was to mirror theoretical descriptions



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Description in paper

Algorithm 3.1: CalcCoin Input: $pk_{\mathcal{M}} \in \{0,1\}^*$ merchant's public key, $contract \in \{0,1\}^*$ User Data: u private key, g^u public key, (s,t,σ,J) a wallet coin $R \leftarrow H(pk_{\mathcal{M}}||info)$; $S \leftarrow g^{1/(s+x+1)}$; $T \leftarrow g^u(g^{1/(t+x+1)})^R$; Calculate ZKPOK Φ of (J, u, s, t, σ) such that: $0 \le J < n$ $S = g^{1/(s+x+1)}$ $T = g^u(g^{1/(t+x+1)})^R$ VerifySig $(pk_{\mathcal{B}}, (u, s, t), \sigma) = true$ return (S, T, Φ, R)

Step 1: writing programs in ZKPDL

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Description in ZKPDL

computation:

```
compute:
    S := g^(1/(s+x+1))
    T := g^u * (g^(1/(t+x+1)))^R
proof:
    given:
    group: G = <g,h>
    elements in G: S, T
prove knowledge of:
    exponents in G: u,s,t,x
    integer: J
such that:
    range: 0 <= J < n
    S = g^(1/(s+x+1))
```

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Description in ZKPDL

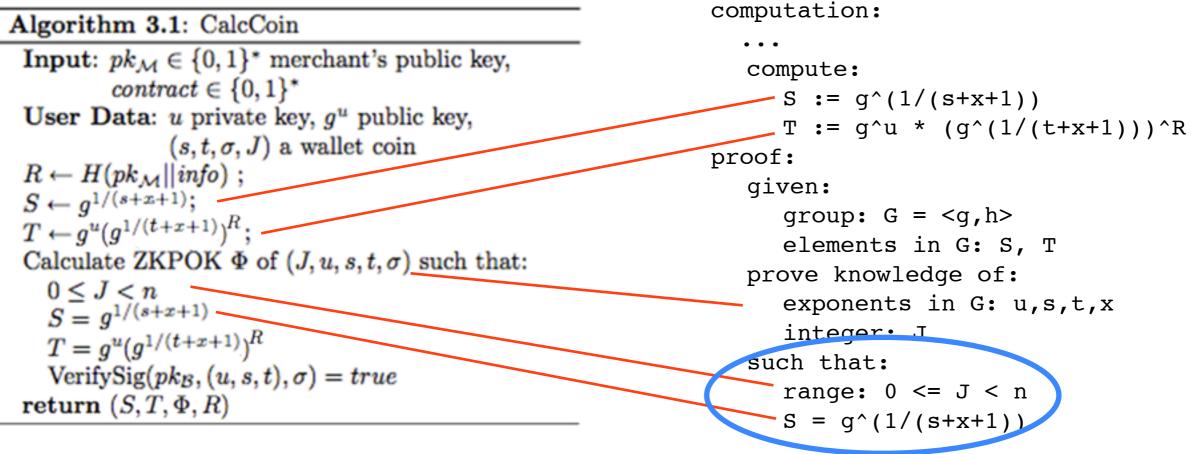
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Step 1: writing programs in ZKPDL

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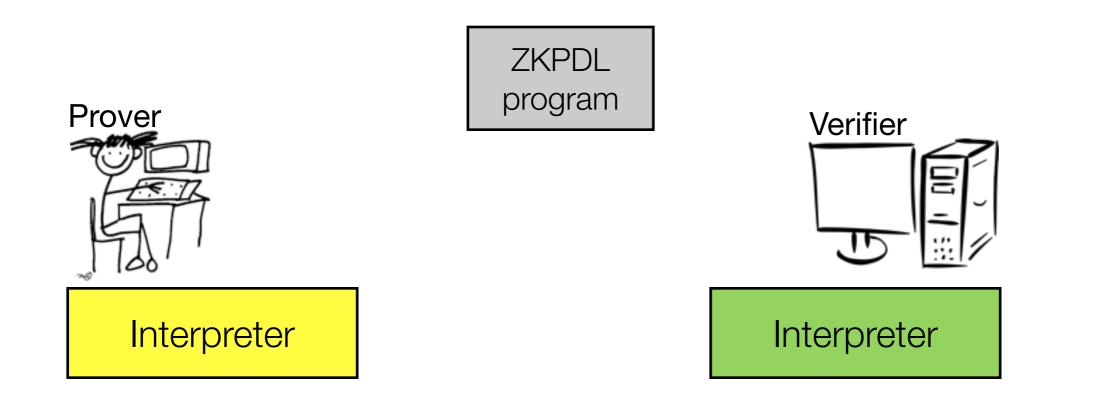


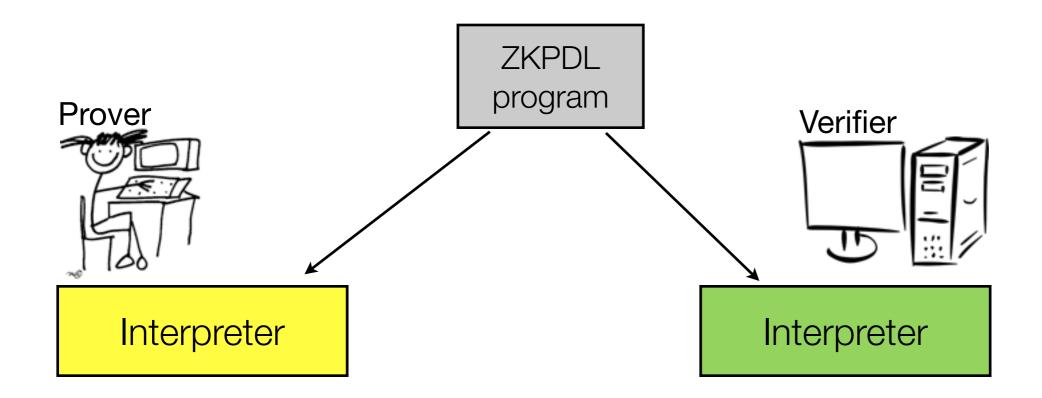
Currently support four ZKP types, enough for vast majority of applications

Should also be easy to add new types if they're needed

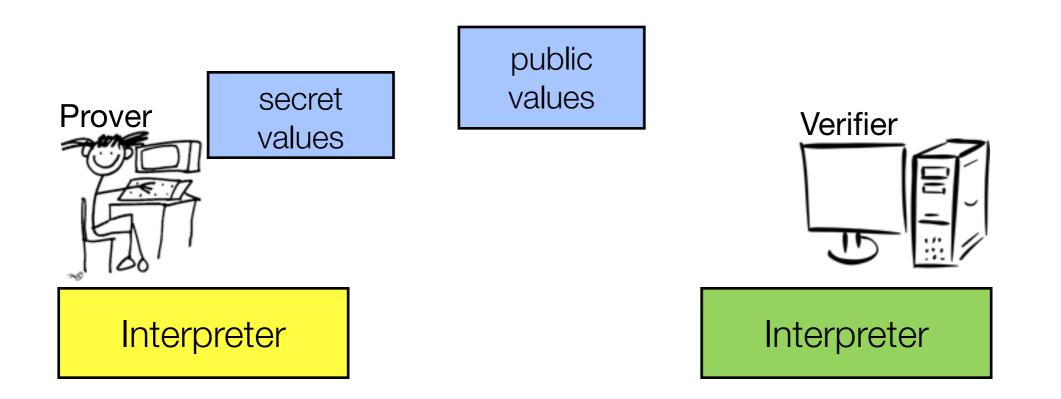




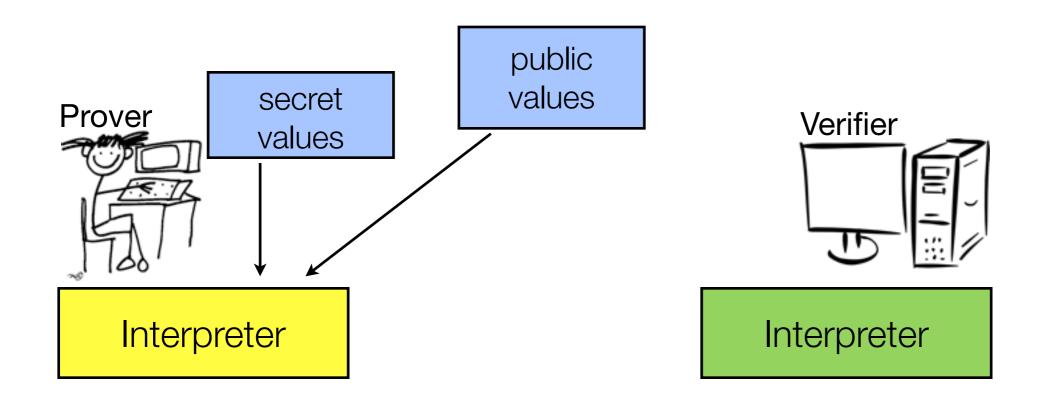




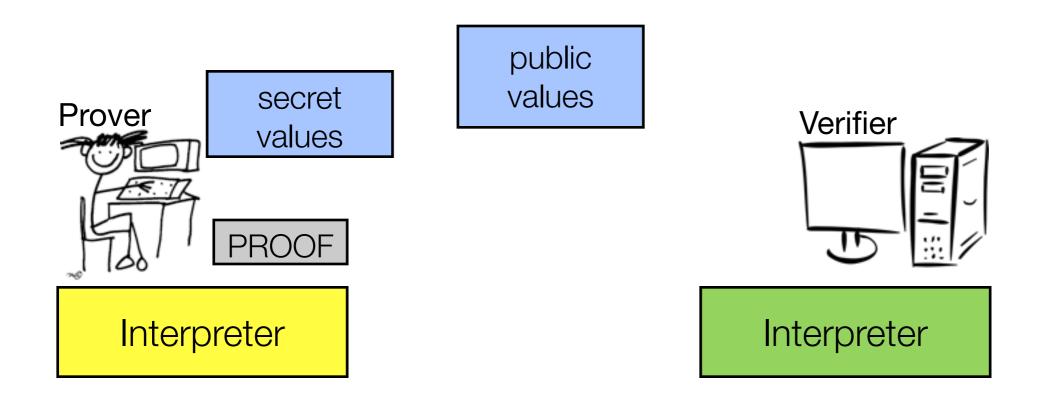
• At compile time, check program syntax, types, etc.



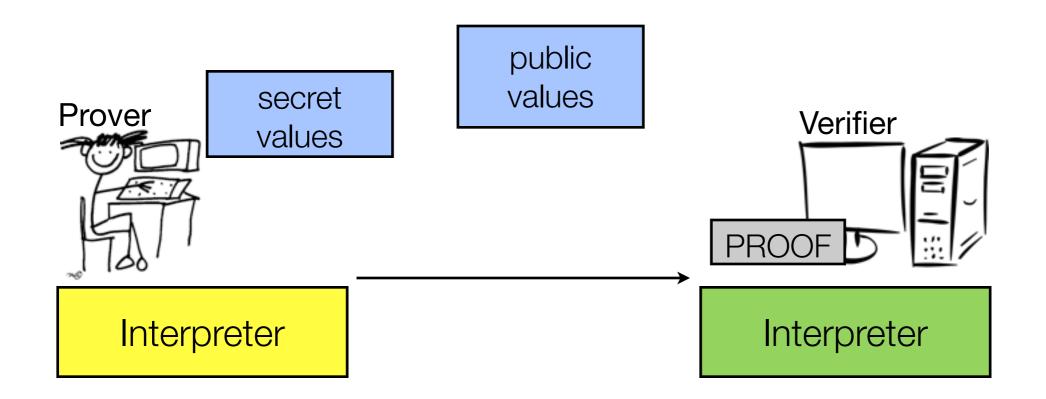
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- At run time, need all values to be proved



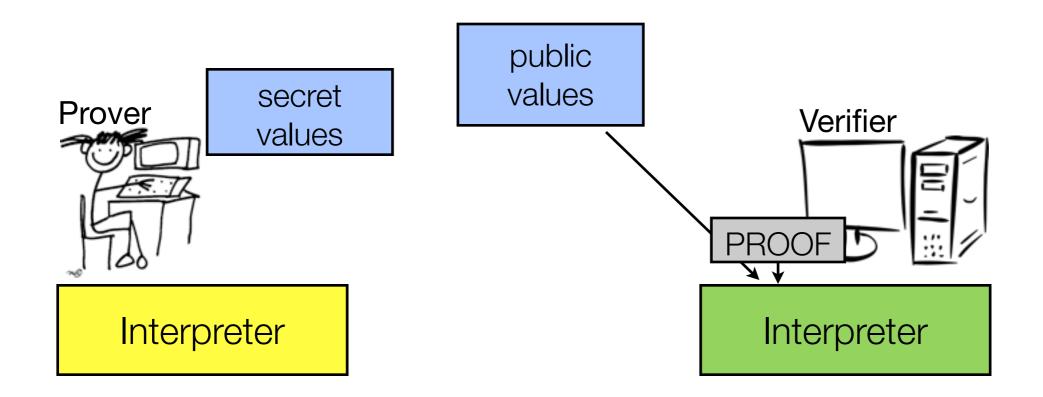
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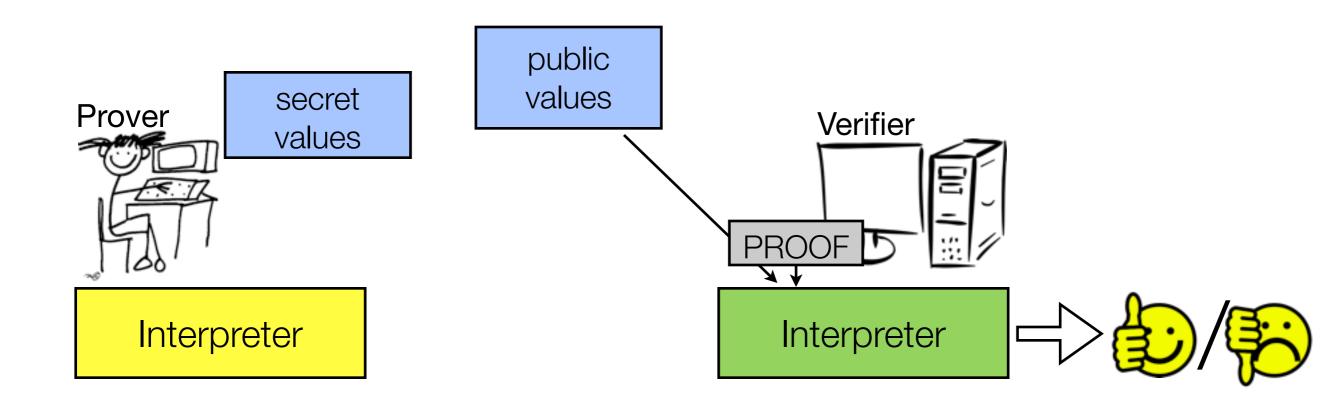
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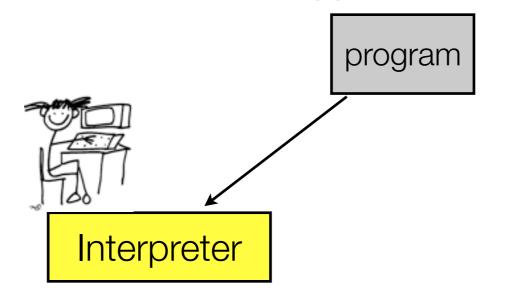


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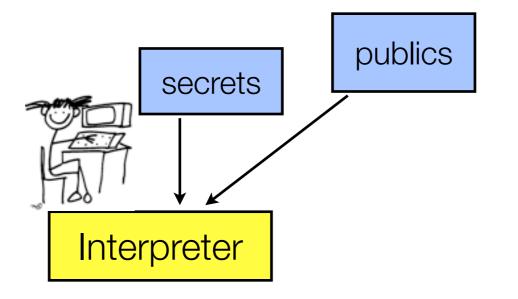




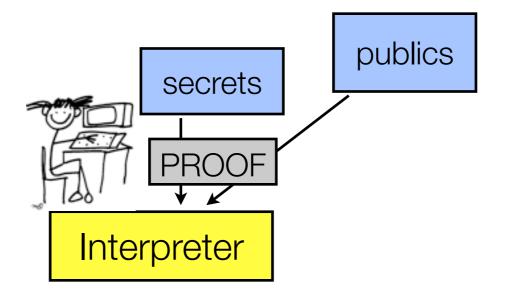








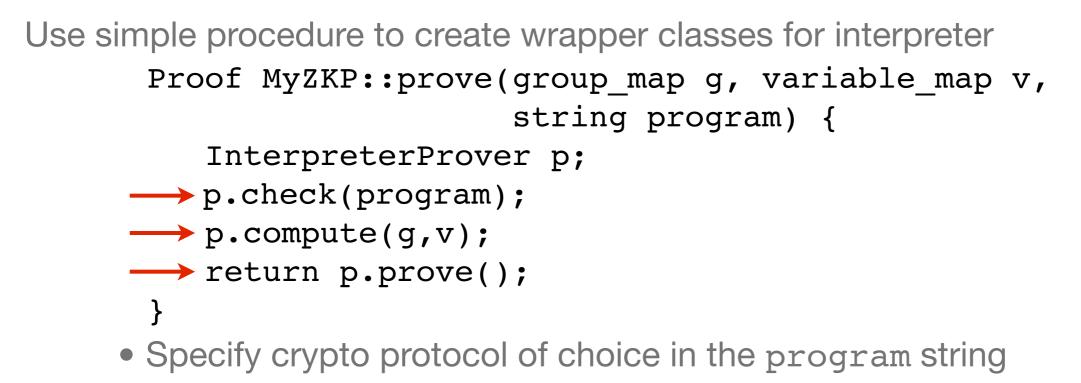






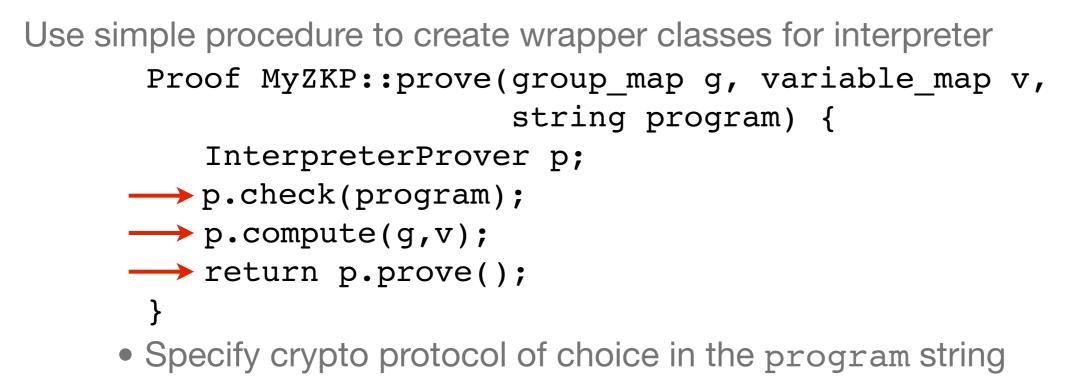






• Feed numeric values in and you're done!

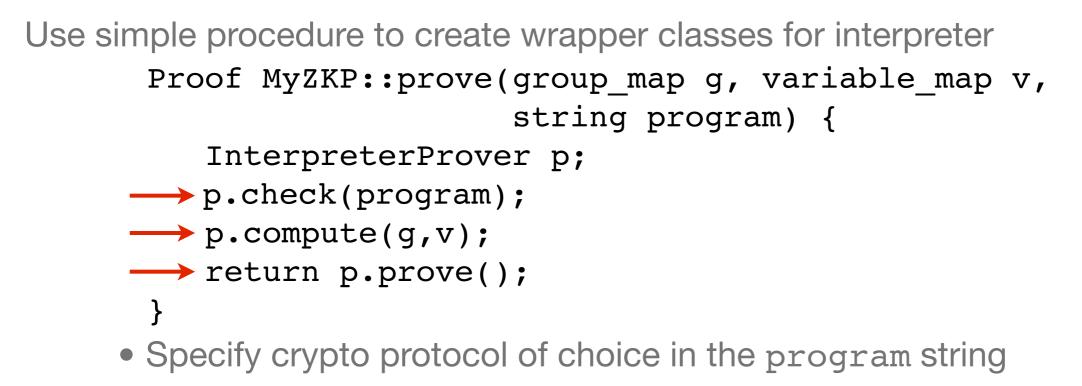




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Solves issues of reusability and of time





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Solves issues of reusability and of time

Took 3-4 months to build interpreter, then one month to reconstruct library

In addition to usability, can achieve improvements in efficiency

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Have optimizations built into the interpreter

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Have optimizations built into the interpreter

• Cache powers of bases used for modular exponentiation

Often have g^x*h^r mod N, numbers are 1000 bits long!

Use common single- and multi-exponentiation techniques

In addition to usability, can achieve improvements in efficiency

Have optimizations built into the interpreter

• Cache powers of bases used for modular exponentiation

Often have g^x*h^r mod N, numbers are 1000 bits long!

Use common single- and multi-exponentiation techniques

• Save copy of interpreter state after compilation

Did caching help?

| Program type | Prover (ms) | | Verifier (ms) | | Proof size | Cache size | Multi-exps | |
|-----------------------|-------------|---------|---------------|---------|------------|------------|------------|----------|
| | With cache | Without | With cache | Without | (bytes) | (Mbytes) | Prover | Verifier |
| DLR proof | 3.07 | 3.08 | 1.26 | 1.25 | 511 | 0 | 2 | 1 |
| Multiplication proof | 2.03 | 4.07 | 1.66 | 2.32 | 848 | 33.5 | 8 | 2 |
| Range proof | 36.36 | 74.52 | 21.63 | 31.54 | 5455 | 33.5 | 31 | 11 |
| CL recipient proof | 119.92 | 248.31 | 70.76 | 112.13 | 19189 | 134.2 | 104 | 39 |
| CL issuer proof | 7.29 | 7.38 | 1.73 | 1.73 | 1097 | 0 | 2 | 1 |
| CL possession proof | 125.89 | 253.17 | 78.19 | 117.67 | 19979 | 134.2 | 109 | 40 |
| Verifiable encryption | 416.09 | 617.61 | 121.87 | 162.77 | 24501 | 190.2 | 113 | 42 |
| Coin | 134.37 | 271.34 | 83.01 | 121.83 | 22526 | 223.7 | 122 | 45 |

On the prover side, saw about a 50% speed-up using all optimizations

On the verifier side, about 30% (less computation)

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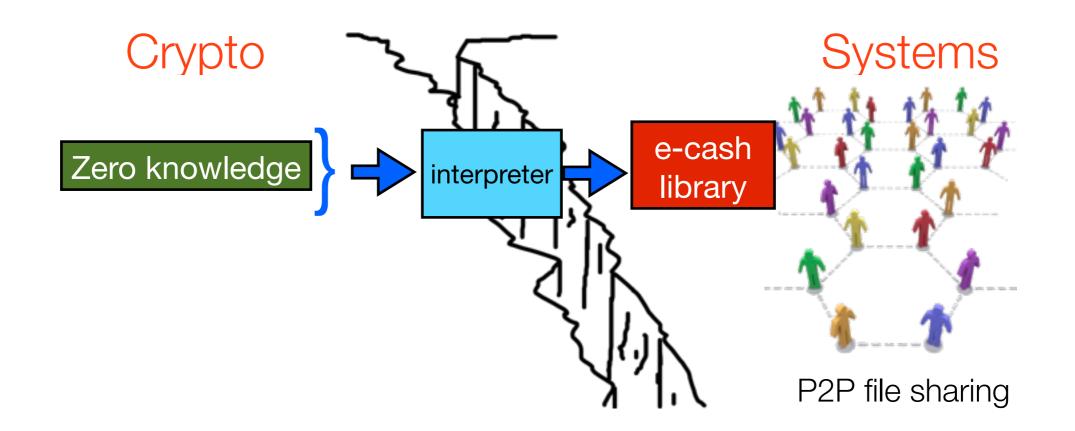
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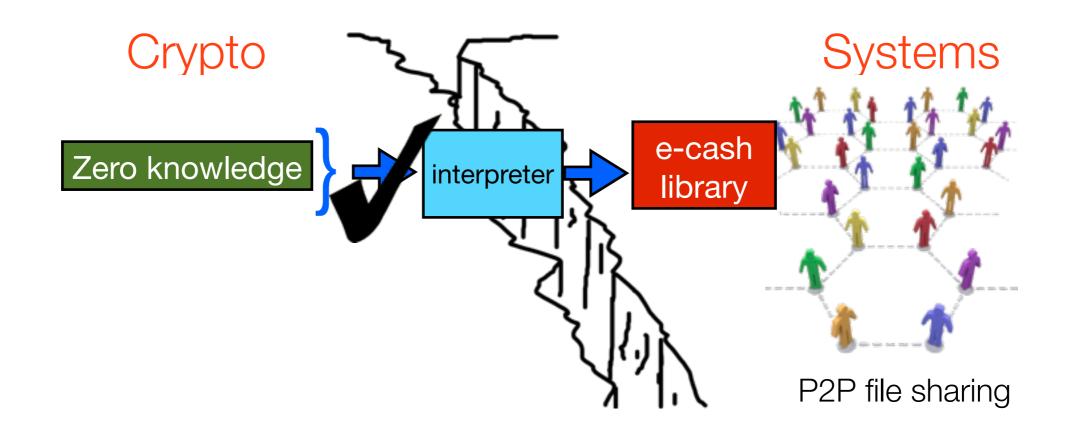
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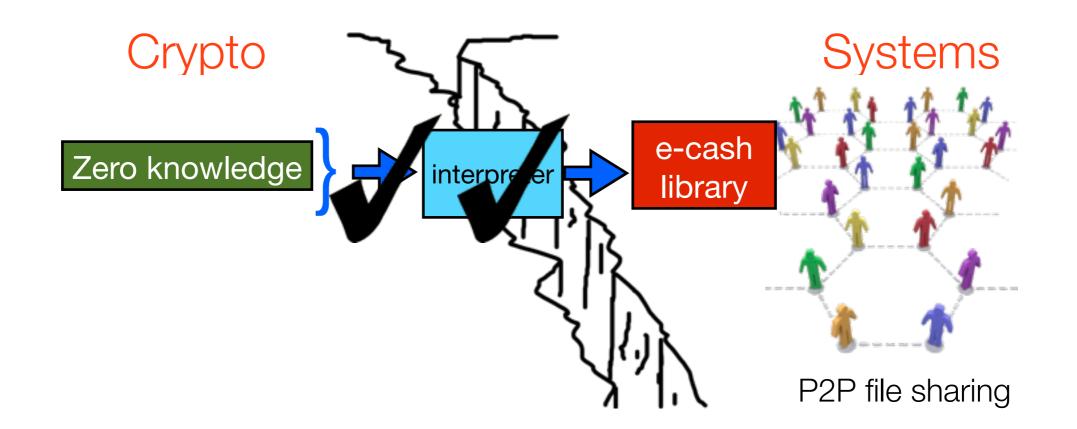
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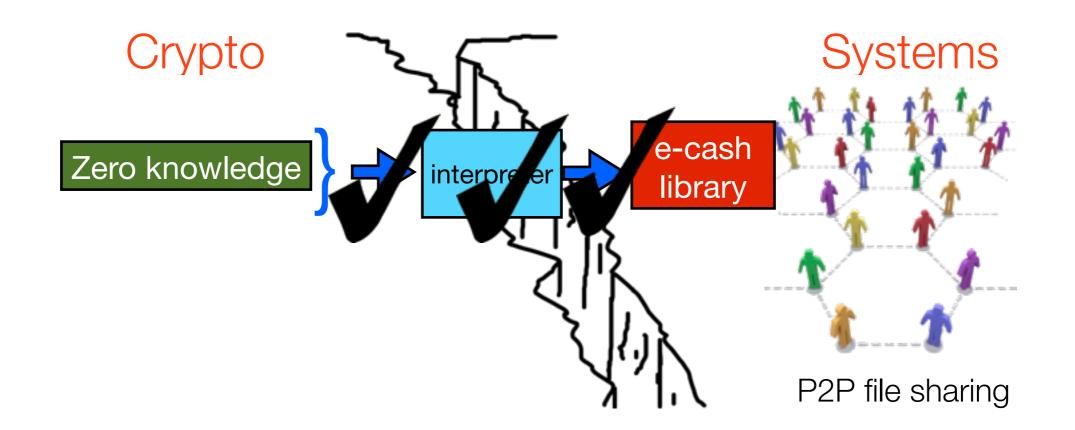
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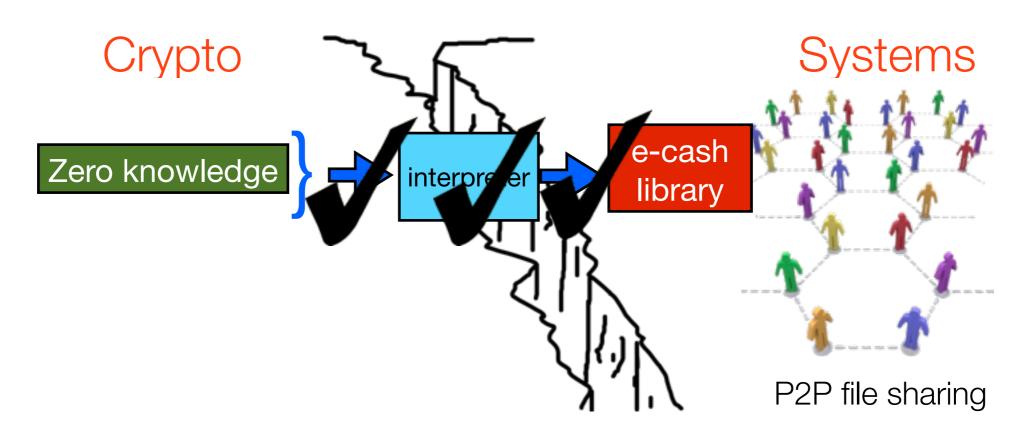
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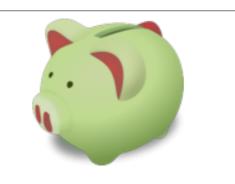


E-cash was originally developed [Ch82] as replacement for currency

Now, view e-cash in context of token systems

- Our usage in P2P file-sharing schemes [BCE+07]
- Provides anonymous transportation ticketing (future work)

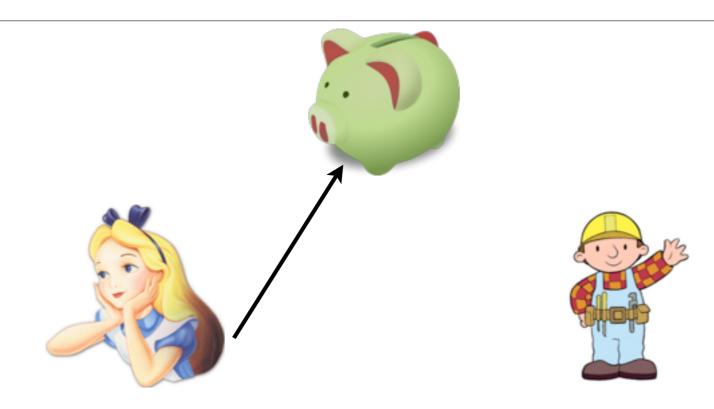
How e-cash works [Ch82, CHL05, CLM07]





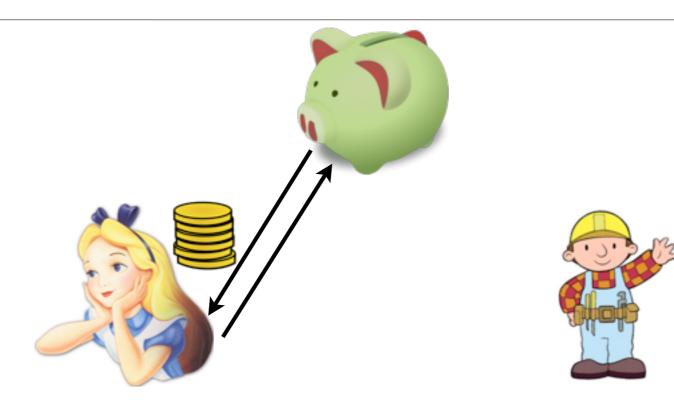


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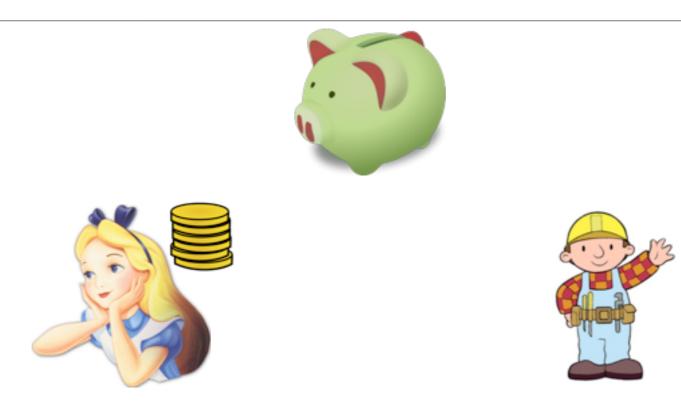


Withdraw: Alice gets coins from bank

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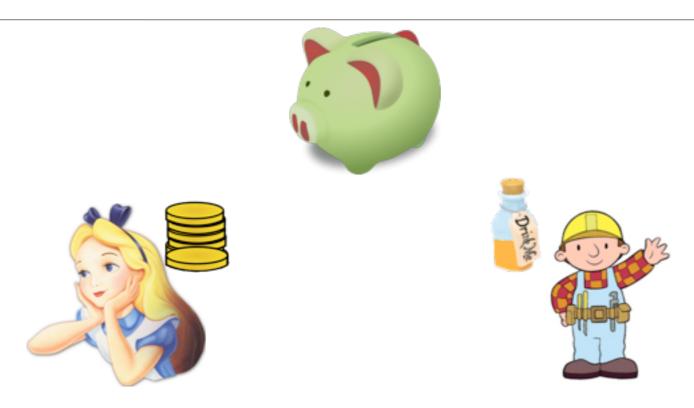


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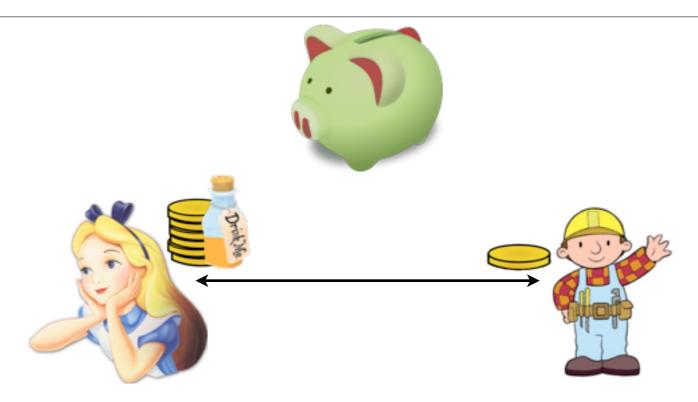
Withdraw: Alice gets coins from bank

Buy: Alice gives Bob coin in exchange for her purchase



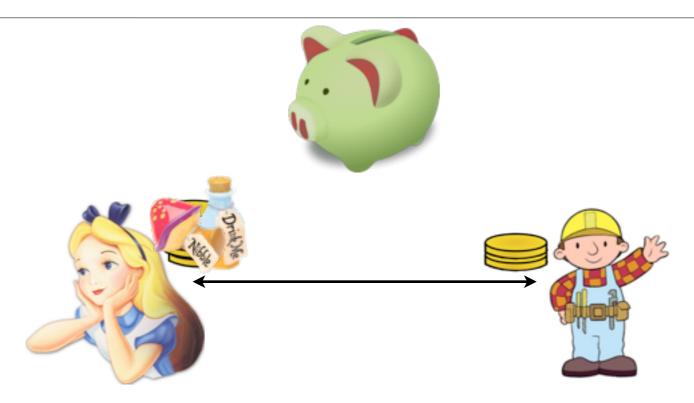
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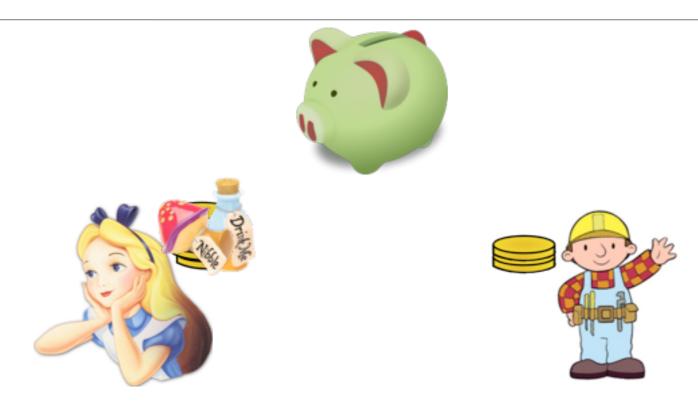
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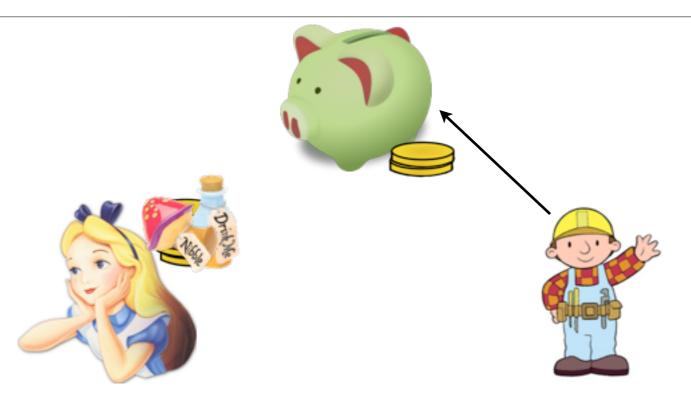


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Deposit: Bob deposits these coins with the bank

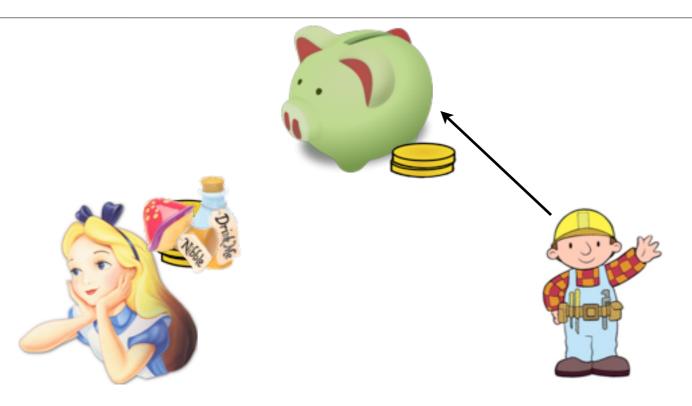


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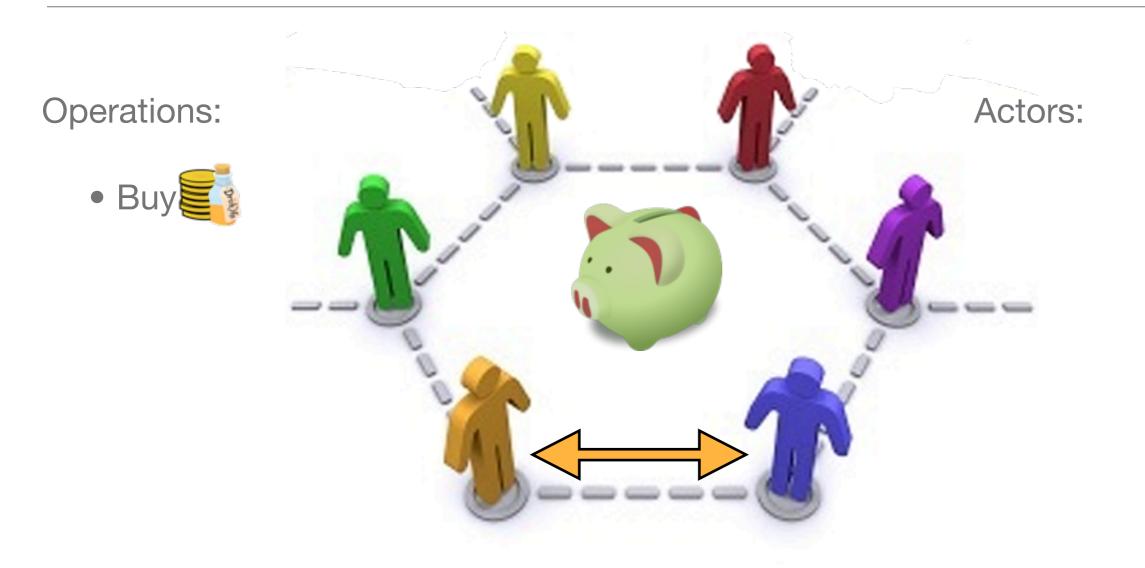
Deposit: Bob deposits these coins with the bank

Untraceability: Bank cannot trace the deposited coins back to Alice

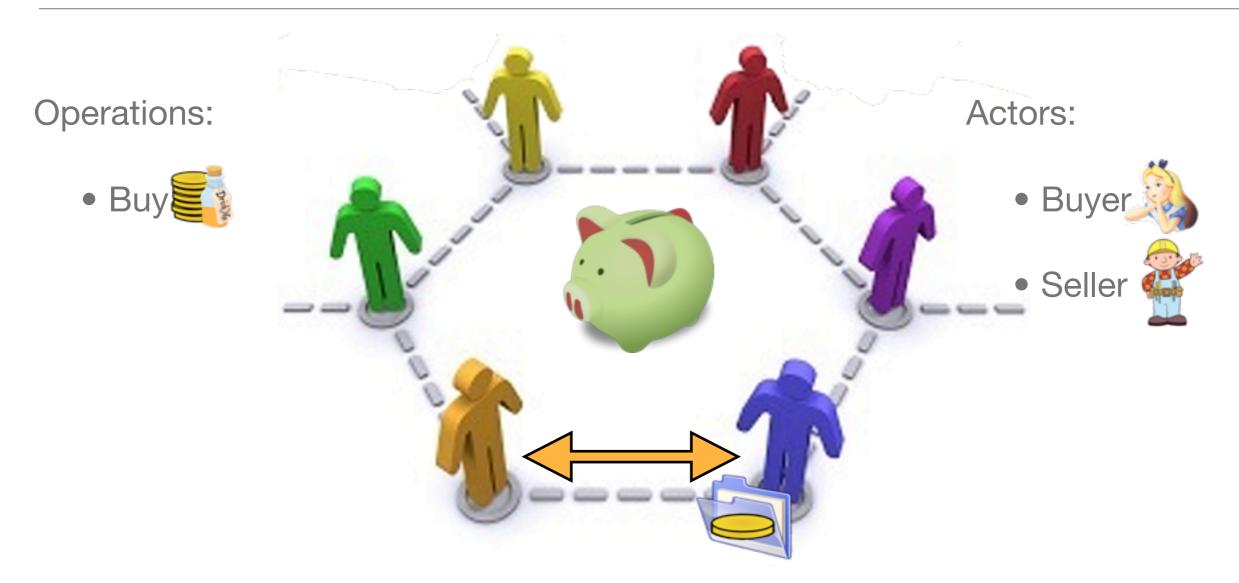












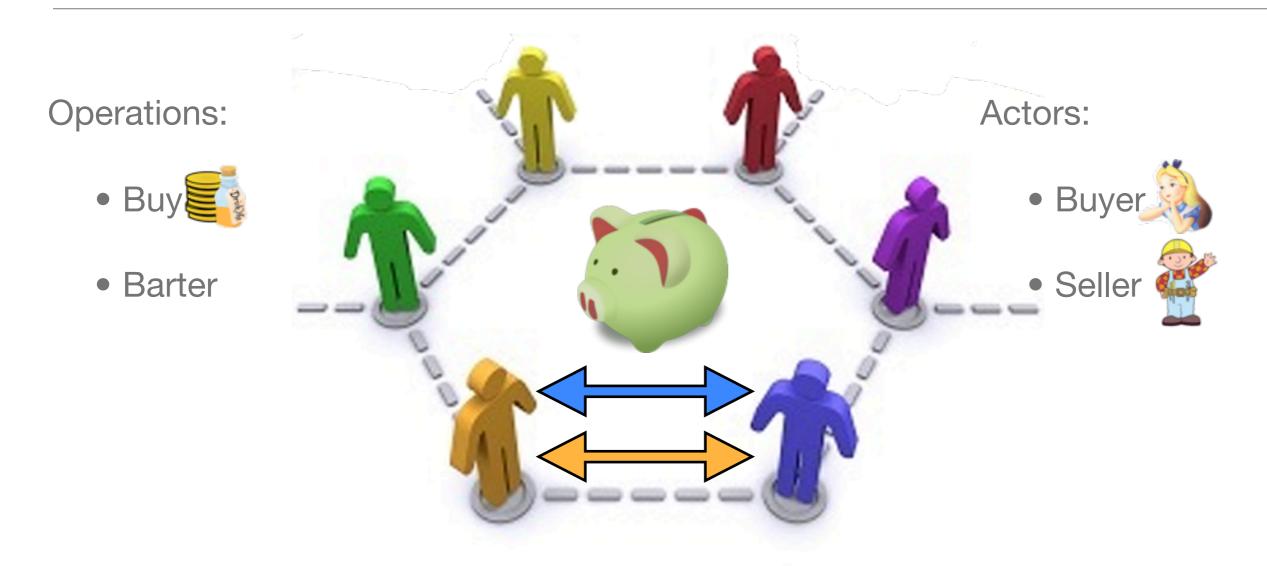




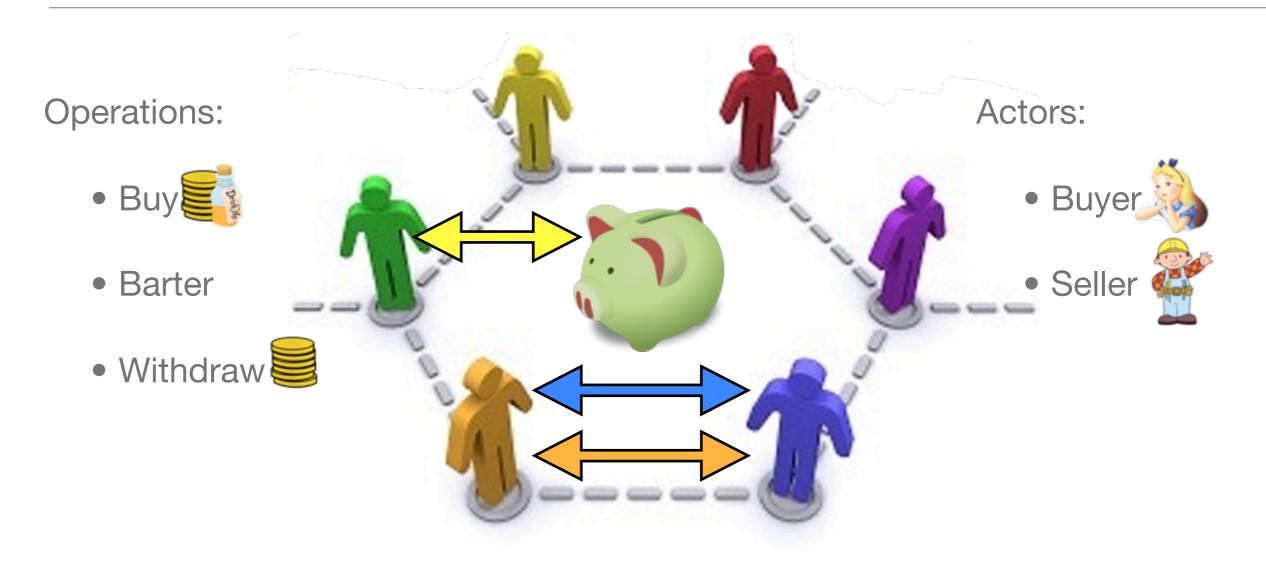




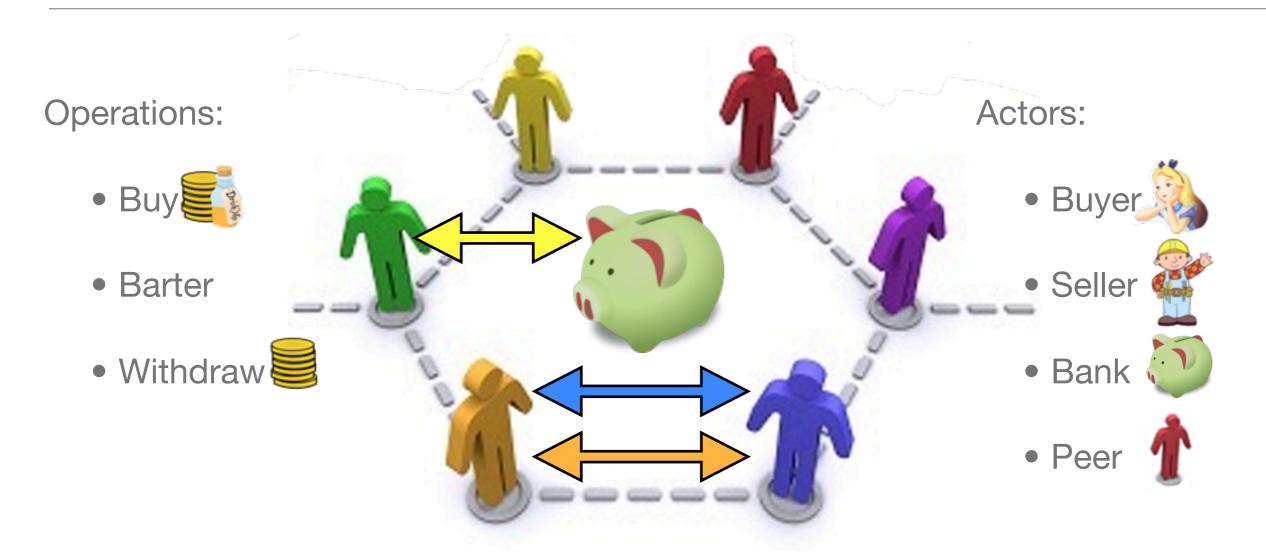
How e-cash can improve P2P interactions:



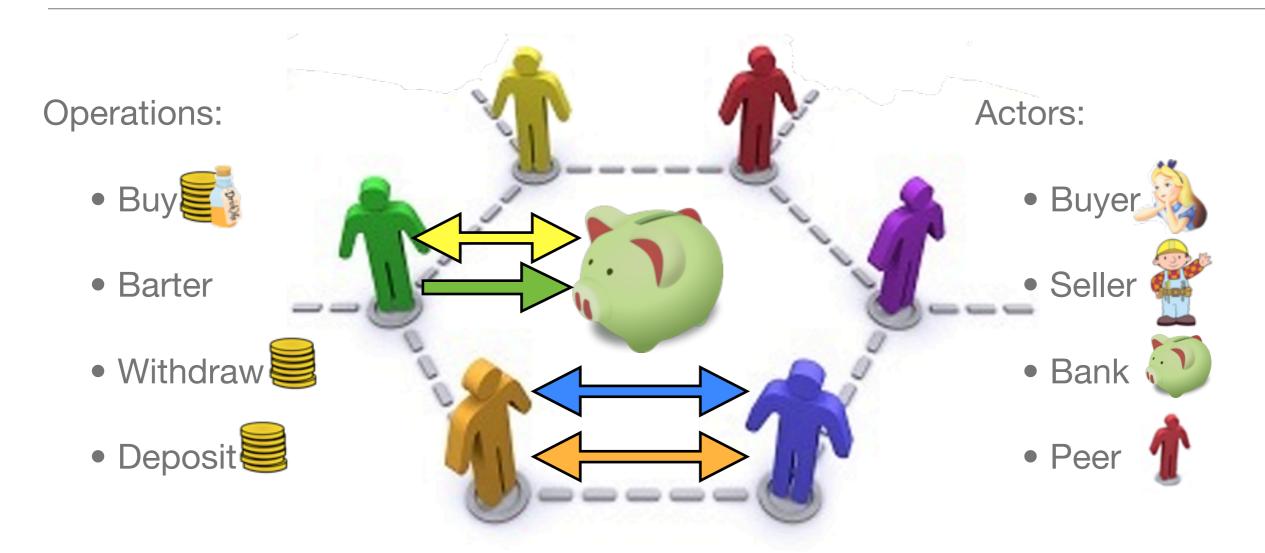
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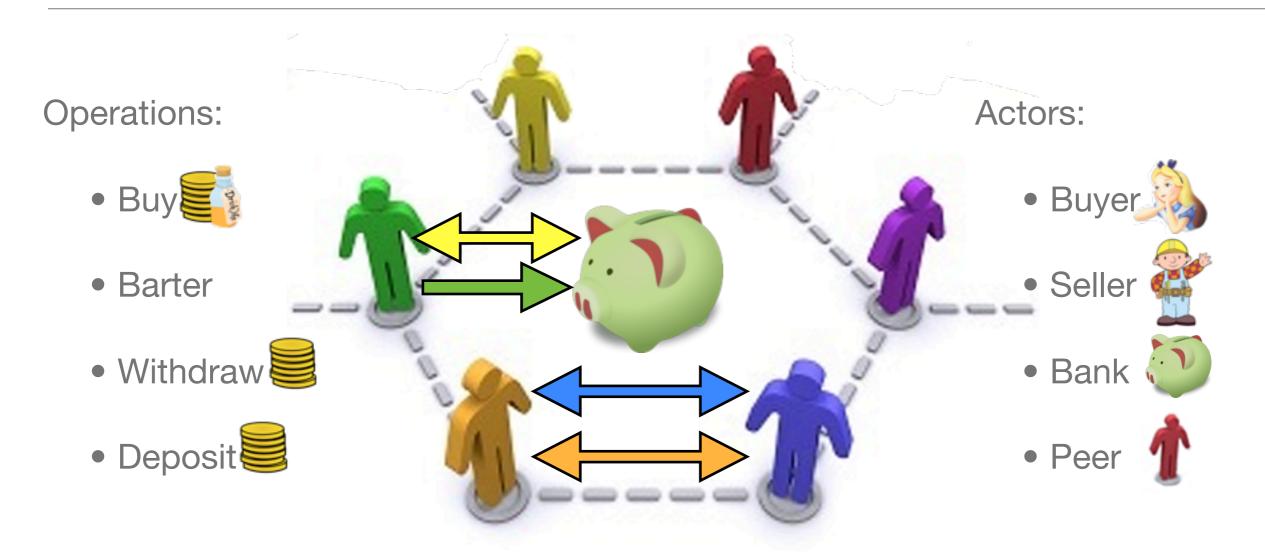
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How e-cash can improve P2P interactions:



- Guarantees fair exchange [BCE+07,KL10] between peers
- Allows bank to monitor upload/download ratio without sacrificing privacy 15

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• As application of zero knowledge, provide library only for e-cash

Idemix project [CH02, BBC+09] provides anonymous credentials

In summary...

- Wrote interpreter to make cryptographer's job easier
 - Demonstrated efficiency and usability
- Wrote library to make programmer's job easier
- All source code and documentation available freely online:

• <u>http://github.com/brownie/cashlib</u>

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Any questions?

Zero knowledge proof types

- What types of proofs do we support?
 - Proof of discrete log representation (DLR): given c, prove c = g^x*h^r
 - Equality of DLR: given c and d, prove c = g^x*h^r and d = g^x*h^s
 - Multiplication: prove x = y*z for secret values x, y, z
 - Range: for secret x and public lo, hi, prove lo <= x < hi